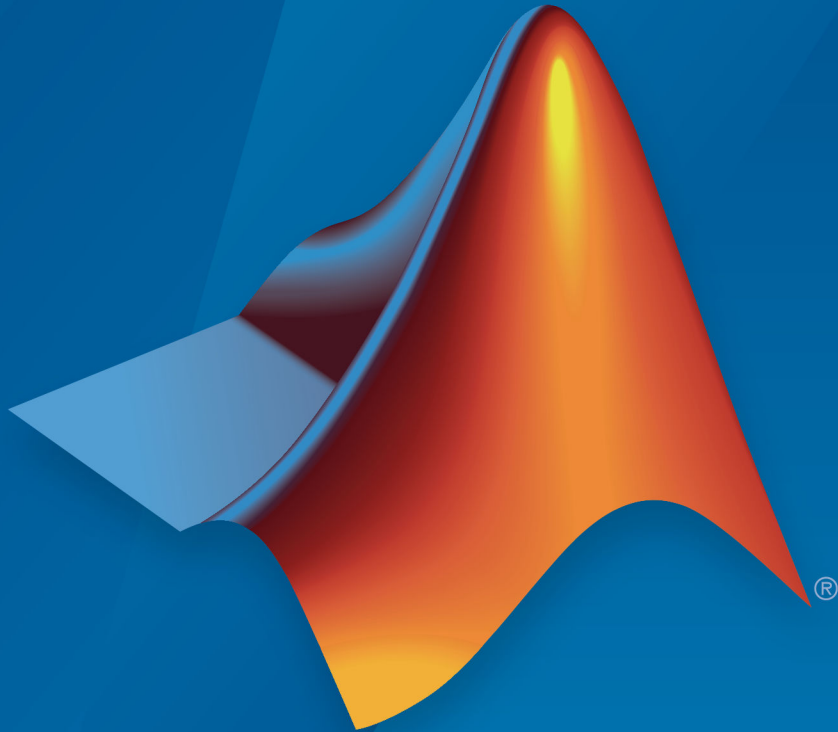


**MATLAB®**

Data Analysis



**MATLAB®**

R2018a



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*MATLAB® Data Analysis*

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## Revision History

September 2005	Online only	New for MATLAB Version 7.1 (Release 14SP3)
March 2006	Online only	Revised for MATLAB Version 7.2 (Release 2006a)
September 2006	Online only	Revised for MATLAB Version 7.3 (Release 2006b)
March 2007	Online only	Revised for MATLAB Version 7.4 (Release 2007a)
September 2007	Online only	Revised for MATLAB Version 7.5 (Release 2007b)
March 2008	Online only	Revised for MATLAB Version 7.6 (Release 2008a)
October 2008	Online only	Revised for MATLAB Version 7.7 (Release 2008b)
March 2009	Online only	Revised for MATLAB 7.8 (Release 2009a)
September 2009	Online only	Revised for MATLAB 7.9 (Release 2009b)
March 2010	Online only	Revised for MATLAB 7.10 (Release 2010a)
September 2010	Online only	Revised for MATLAB Version 7.11 (R2010b)
April 2011	Online only	Revised for MATLAB Version 7.12 (R2011a)
September 2011	Online only	Revised for MATLAB Version 7.13 (R2011b)
March 2012	Online only	Revised for MATLAB Version 7.14 (R2012a)
September 2012	Online only	Revised for MATLAB Version 8.0 (R2012b)
March 2013	Online only	Revised for MATLAB Version 8.1 (R2013a)
September 2013	Online only	Revised for MATLAB Version 8.2 (R2013b)
March 2014	Online only	Revised for MATLAB Version 8.3 (R2014a)
October 2014	Online only	Revised for MATLAB Version 8.4 (R2014b)
March 2015	Online only	Revised for MATLAB Version 8.5 (R2015a)
September 2015	Online only	Revised for MATLAB Version 8.6 (R2015b)
March 2016	Online only	Revised for MATLAB Version 9.0 (R2016a)
September 2016	Online only	Revised for MATLAB Version 9.1 (R2016b)
March 2017	Online only	Revised for MATLAB Version 9.2 (R2017a)
September 2017	Online only	Revised for MATLAB Version 9.3 (R2017b)
March 2018	Online only	Revised for MATLAB Version 9.4 (R2018a)



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# Data Processing

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## Importing and Exporting Data

<b>In this section...</b>
“Importing Data into the Workspace” on page 1-2
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### Importing Data into the Workspace

The first step in analyzing data is to import it into the MATLAB workspace. See “Methods for Importing Data” for information about importing data from specific file formats.

### Exporting Data from the Workspace

When you analyze your data, you might create new variables or modified imported variables. You can export variables from the MATLAB workspace to various file formats, both character-based and binary. You can, for example, create HDF and Microsoft® Excel® files containing your data. For details, see the documentation on “Supported File Formats for Import and Export”.

# Plotting Data

## In this section...

“Introduction” on page 1-3

“Load and Plot Data from Text File” on page 1-3

## Introduction

After you import data into the MATLAB workspace, it is a good idea to plot the data so that you can explore its features. An exploratory plot of your data enables you to identify discontinuities and potential outliers, as well as the regions of interest.

The MATLAB figure window displays plots. See “Types of MATLAB Plots” for a full description of the figure window. It also discusses the various interactive tools available for editing and customizing MATLAB graphics.

## Load and Plot Data from Text File

This example uses sample data in `count.dat`, a space-delimited text file. The file consists of three sets of hourly traffic counts, recorded at three different town intersections over a 24-hour period. Each data column in the file represents data for one intersection.

### Load the `count.dat` Data

Import data into the workspace using the `load` function.

```
load count.dat
```

Loading this data creates a 24-by-3 matrix called `count` in the MATLAB workspace.

Get the size of the data matrix.

```
[n,p] = size(count)
```

```
n = 24
```

```
p = 3
```

`n` represents the number of rows, and `p` represents the number of columns.

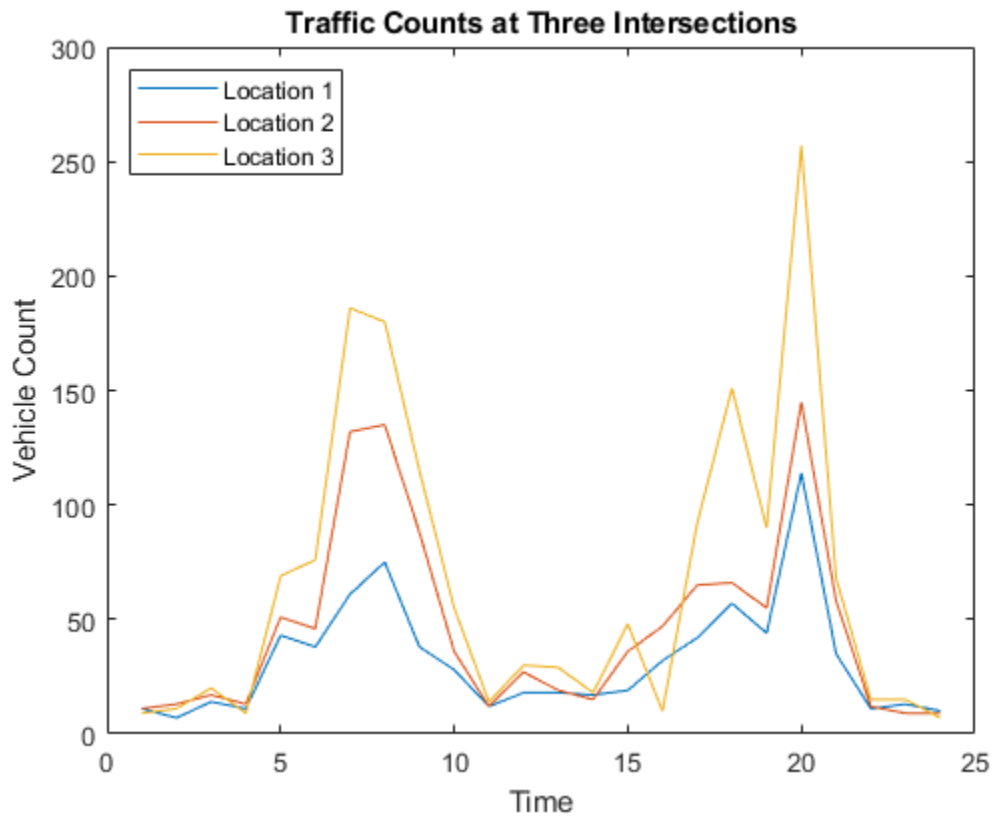
## Plot the count.dat Data

Create a time vector, `t`, containing integers from 1 to `n`.

```
t = 1:n;
```

Plot the data as a function of time, and annotate the plot.

```
plot(t,count),  
legend('Location 1','Location 2','Location 3','Location','NorthWest')  
xlabel('Time'), ylabel('Vehicle Count')  
title('Traffic Counts at Three Intersections')
```



## See Also

[legend](#) | [load](#) | [plot](#) | [size](#) | [title](#) | [xlabel](#) | [ylabel](#)

## More About

- [“Types of MATLAB Plots”](#)

## Missing Data in MATLAB

Working with missing data is a common task in data preprocessing. Although sometimes missing values signify a meaningful event in the data, they often represent unreliable or unusable data points. In either case, MATLAB has many options for handling missing data.

### Create and Organize Missing Data

The form that missing values take in MATLAB depends on the data type. For example, numeric data types such as `double` use `NaN` (not a number) to represent missing values.

```
x = [NaN 1 2 3 4];
```

You can also use the missing value to represent missing numeric data or data of other types, such as `datetime`, `string`, and `categorical`. MATLAB automatically converts the missing value to the data's native type.

```
xDouble = [missing 1 2 3 4]
```

```
xDouble = 1x5
```

```
NaN    1    2    3    4
```

```
xDatetime = [missing datetime(2014,1:4,1)]
```

```
xDatetime = 1x5 datetime array
```

```
Columns 1 through 3
```

```
NaT                01-Jan-2014 00:00:00    01-Feb-2014 00:00:00
```

```
Columns 4 through 5
```

```
01-Mar-2014 00:00:00    01-Apr-2014 00:00:00
```

```
xString = [missing "a" "b" "c" "d"]
```

```
xString = 1x5 string array
```

```
<missing>    "a"    "b"    "c"    "d"
```

```
xCategorical = [missing categorical({'cat1' 'cat2' 'cat3' 'cat4'})]
```

```
xCategorical = 1x5 categorical array
<undefined>   cat1   cat2   cat3   cat4
```

A data set might contain values that you want to treat as missing data, but are not standard MATLAB missing values in MATLAB such as NaN. You can use the `standardizeMissing` function to convert those values to the standard missing value for that data type. For example, treat 4 as a missing double value in addition to NaN.

```
xStandard = standardizeMissing(xDouble,[4 NaN])
```

```
xStandard = 1x5
   NaN     1     2     3   NaN
```

Suppose you want to keep missing values as part of your data set but segregate them from the rest of the data. Several MATLAB functions enable you to control the placement of missing values before further processing. For example, use the `'MissingPlacement'` option with the `sort` function to move NaNs to the end of the data.

```
xSort = sort(xStandard,'MissingPlacement','last')
```

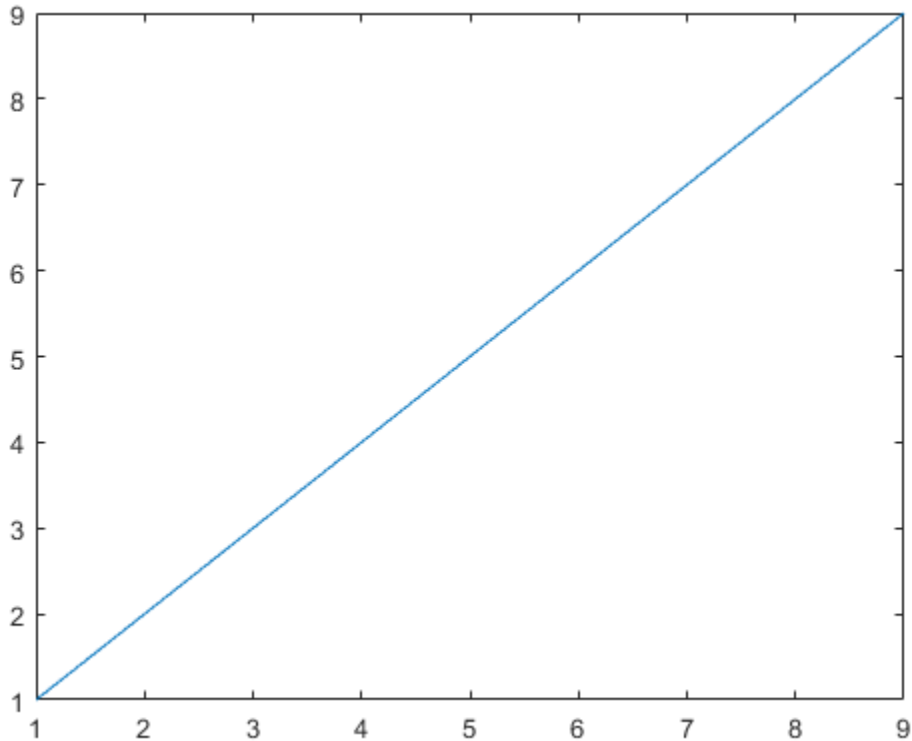
```
xSort = 1x5
     1     2     3   NaN   NaN
```

### Find, Replace, and Ignore Missing Data

Even if you do not explicitly create missing values in MATLAB, they can appear when importing existing data or computing with the data. If you are not aware of missing values in your data, subsequent computation or analysis can be misleading.

For example, if you unknowingly plot a vector containing a NaN value, the NaN does not appear because the `plot` function ignores it and plots the remaining points normally.

```
nanData = [1:9 NaN];
plot(1:10,nanData)
```



However, if you compute the average of the data, the result is `NaN`. In this case, it is more helpful to know in advance that the data contains a `NaN`, and then choose to ignore or remove it before computing the average.

```
meanData = mean(nanData)
```

```
meanData = NaN
```

One way to find `NaN`s in data is by using the `isnan` function, which returns a logical array indicating the location of any `NaN` value.

```
TF = isnan(nanData)
```

```
TF = 1x10 logical array
```



```
0 0 0 0 0 0 0 0 0 1
```

Similarly, the `ismissing` function returns the location of missing values in data for multiple data types.

```
TFdouble = ismissing(xDouble)
```

```
TFdouble = 1x5 logical array
```

```
1 0 0 0 0
```

```
TFdatetime = ismissing(xDatetime)
```

```
TFdatetime = 1x5 logical array
```

```
1 0 0 0 0
```

Suppose you are working with a table or timetable made up of variables with multiple data types. You can find all of the missing values with one call to `ismissing`, regardless of their type.

```
xTable = table(xDouble',xDatetime',xString',xCategorical')
```

```
xTable=5x4 table
```

Var1	Var2	Var3	Var4
NaN	NaT	<missing>	<undefined>
1	01-Jan-2014 00:00:00	"a"	cat1
2	01-Feb-2014 00:00:00	"b"	cat2
3	01-Mar-2014 00:00:00	"c"	cat3
4	01-Apr-2014 00:00:00	"d"	cat4

```
TF = ismissing(xTable)
```

```
TF = 5x4 logical array
```

```
1 1 1 1
0 0 0 0
0 0 0 0
0 0 0 0
```

```
0 0 0 0
```

Missing values can represent unusable data for processing or analysis. Use `fillmissing` to replace missing values with another value, or use `rmmissing` to remove missing values altogether.

```
xFill = fillmissing(xStandard, 'constant', 0)
```

```
xFill = 1x5
```

```
0 1 2 3 0
```

```
xRemove = rmmissing(xStandard)
```

```
xRemove = 1x3
```

```
1 2 3
```

Many MATLAB functions enable you to ignore missing values, without having to explicitly locate, fill, or remove them first. For example, if you compute the sum of a vector containing NaN values, the result is NaN. However, you can directly ignore NaNs in the sum by using the `'omitnan'` option with the `sum` function.

```
sumNan = sum(xDouble)
```

```
sumNan = NaN
```

```
sumOmitnan = sum(xDouble, 'omitnan')
```

```
sumOmitnan = 10
```

## See Also

`ismissing` | `fillmissing` | `standardizeMissing` | `missing`

## Related Examples

- “Clean Messy and Missing Data in Tables”

## Data Smoothing and Outlier Detection

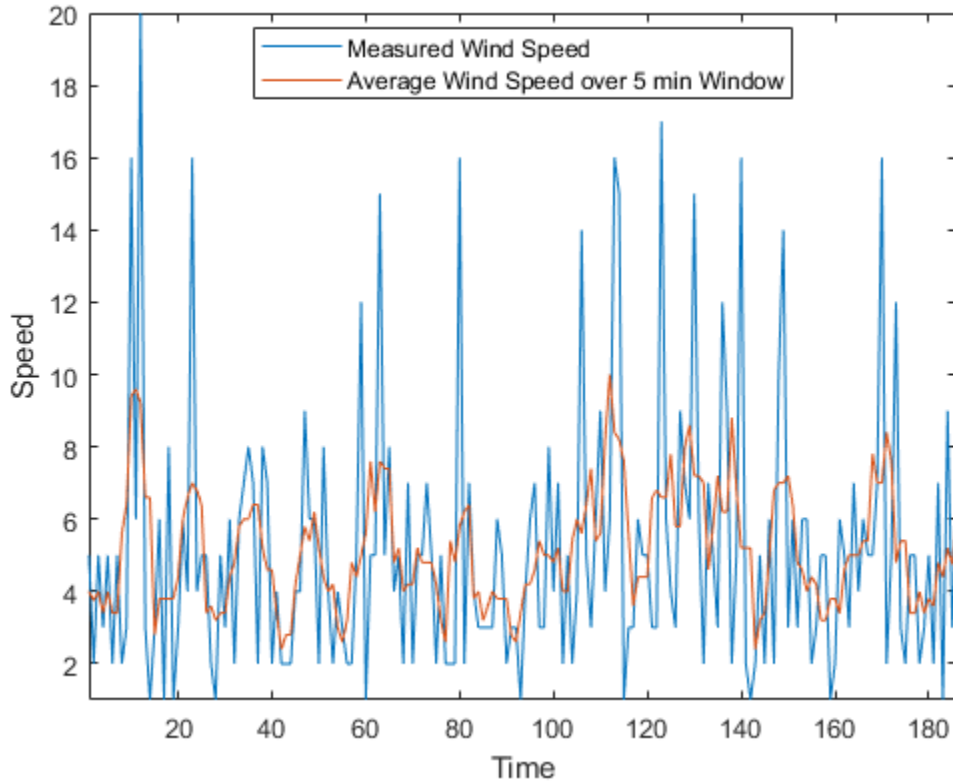
Data smoothing refers to techniques for eliminating unwanted noise or behaviors in data, while outlier detection identifies data points that are significantly different from the rest of the data.

### Moving Window Methods

Moving window methods are ways to process data in smaller batches at a time, typically in order to statistically represent a neighborhood of points in the data. The moving average is a common data smoothing technique that slides a window along the data, computing the mean of the points inside of each window. This can help to eliminate insignificant variations from one data point to the next.

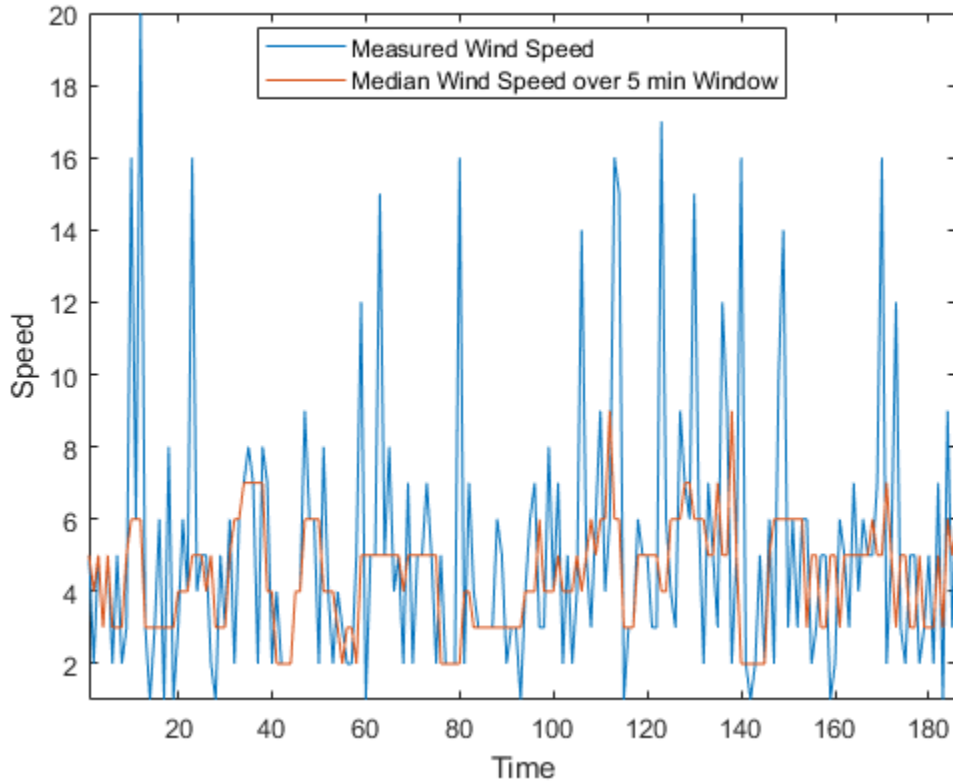
For example, consider wind speed measurements taken every minute for about 3 hours. Use the `movmean` function with a window size of 5 minutes to smooth out high-speed wind gusts.

```
load windData.mat
mins = 1:length(speed);
window = 5;
meanspeed = movmean(speed,window);
plot(mins,speed,mins,meanspeed)
axis tight
legend('Measured Wind Speed','Average Wind Speed over 5 min Window','location','best')
xlabel('Time')
ylabel('Speed')
```



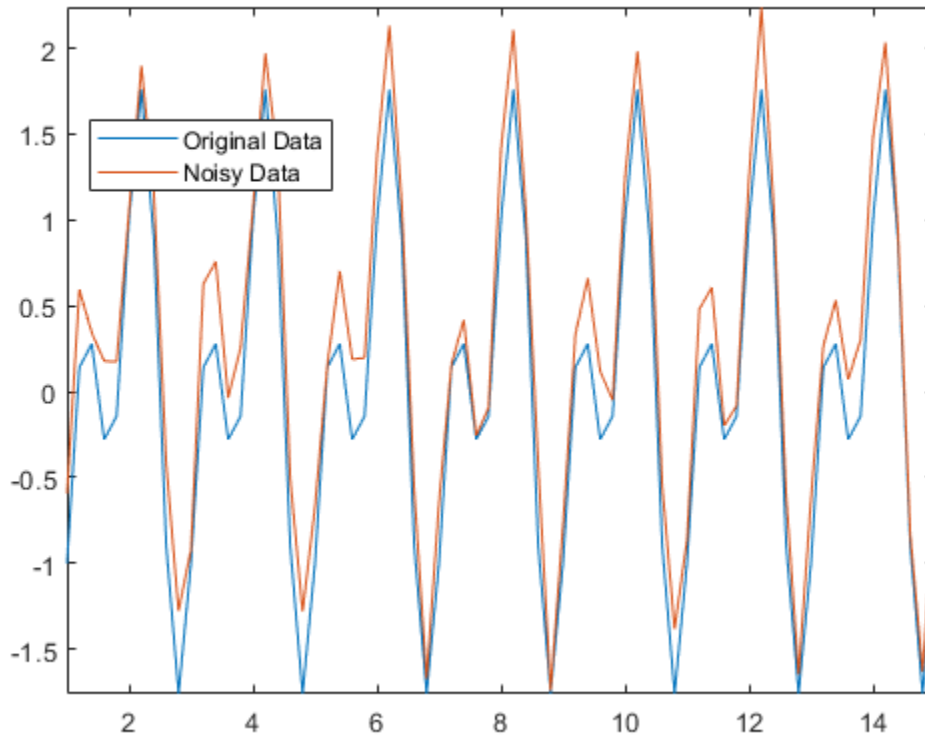
Similarly, you can compute the median wind speed over a sliding window using the `movmedian` function.

```
medianspeed = movmedian(speed>window);  
plot(mins,speed,mins,medianspeed)  
axis tight  
legend('Measured Wind Speed','Median Wind Speed over 5 min Window','location','best')  
xlabel('Time')  
ylabel('Speed')
```



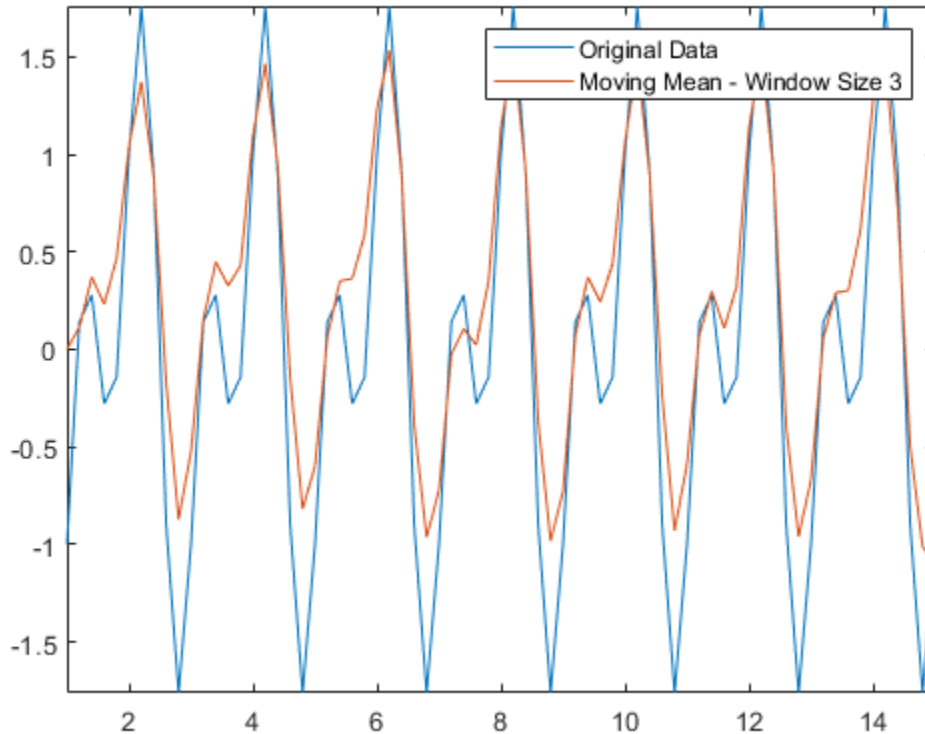
Not all data is suitable for smoothing with a moving window method. For example, create a sinusoidal signal with injected random noise.

```
t = 1:0.2:15;  
A = sin(2*pi*t) + cos(2*pi*0.5*t);  
Anoise = A + 0.5*rand(1,length(t));  
plot(t,A,t,Anoise)  
axis tight  
legend('Original Data','Noisy Data','location','best')
```



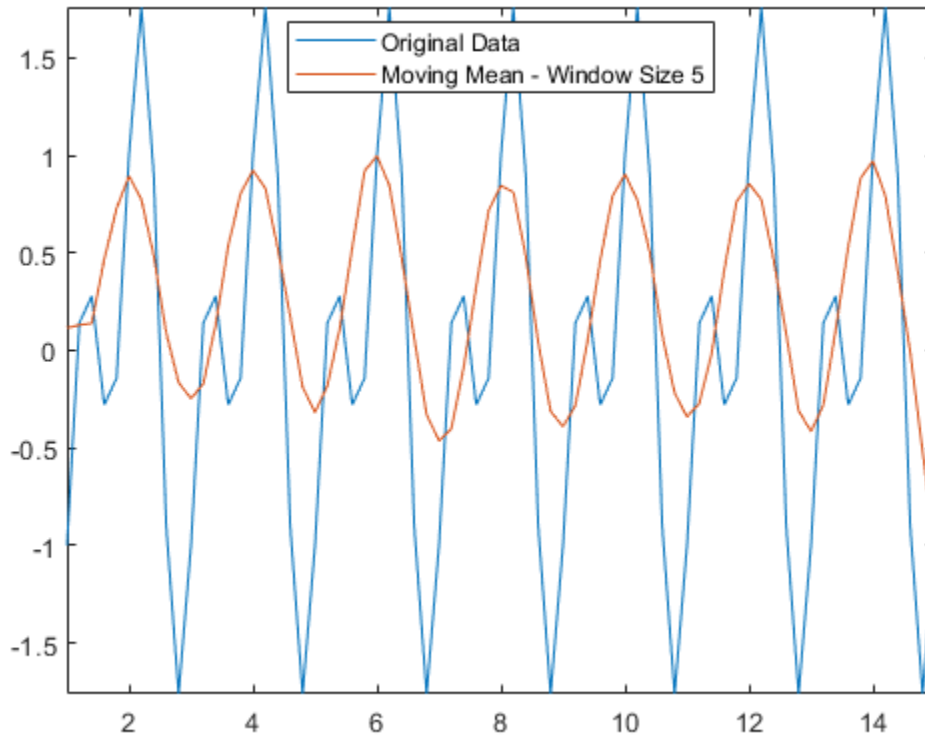
Use a moving mean with a window size of 3 to smooth the noisy data.

```
window = 3;  
Amean = movmean(Anoise,window);  
plot(t,A,t,Amean)  
axis tight  
legend('Original Data','Moving Mean - Window Size 3')
```



The moving mean achieves the general shape of the data, but doesn't capture the valleys (local minima) very accurately. Since the valley points are surrounded by two larger neighbors in each window, the mean is not a very good approximation to those points. If you make the window size larger, the mean eliminates the shorter peaks altogether. For this type of data, you might consider alternative smoothing techniques.

```
Amean = movmean(Anoise,5);  
plot(t,A,t,Amean)  
axis tight  
legend('Original Data','Moving Mean - Window Size 5','location','best')
```



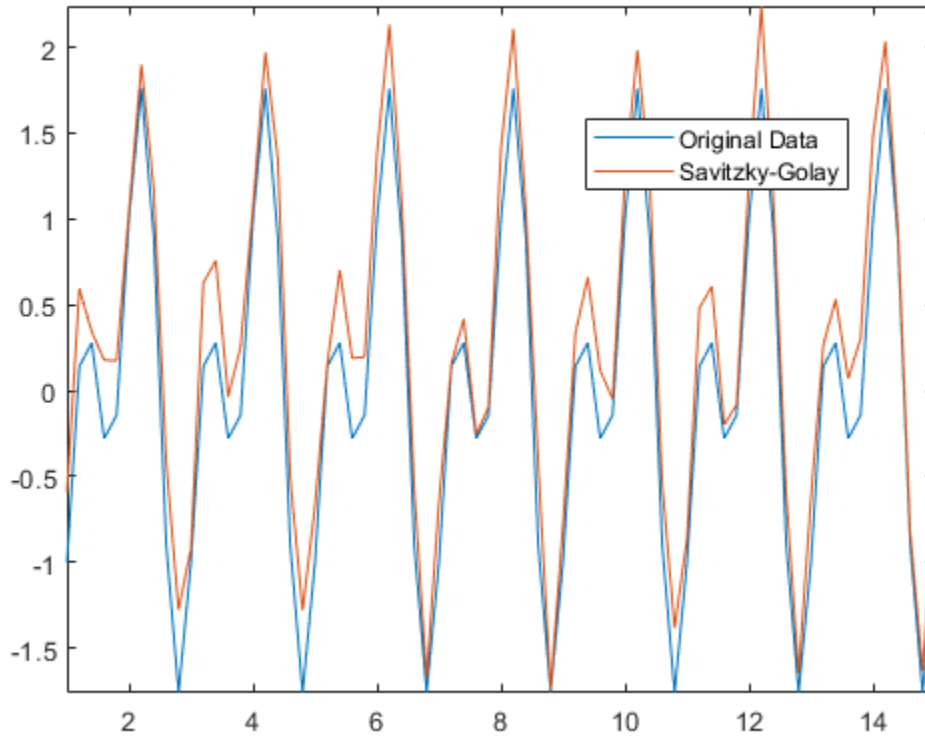
### Common Smoothing Methods

The `smoothdata` function provides several smoothing options such as the Savitzky-Golay method, which is a popular smoothing technique used in signal processing. By default, `smoothdata` chooses a best-guess window size for the method depending on the data.

Use the Savitzky-Golay method to smooth the noisy signal `Anoise`, and output the window size that it uses. This method provides a better valley approximation compared to `movmean`.

```
[Asgolay,window] = smoothdata(Anoise,'sgolay');  
plot(t,A,t,Asgolay)  
axis tight  
legend('Original Data','Savitzky-Golay','location','best')
```



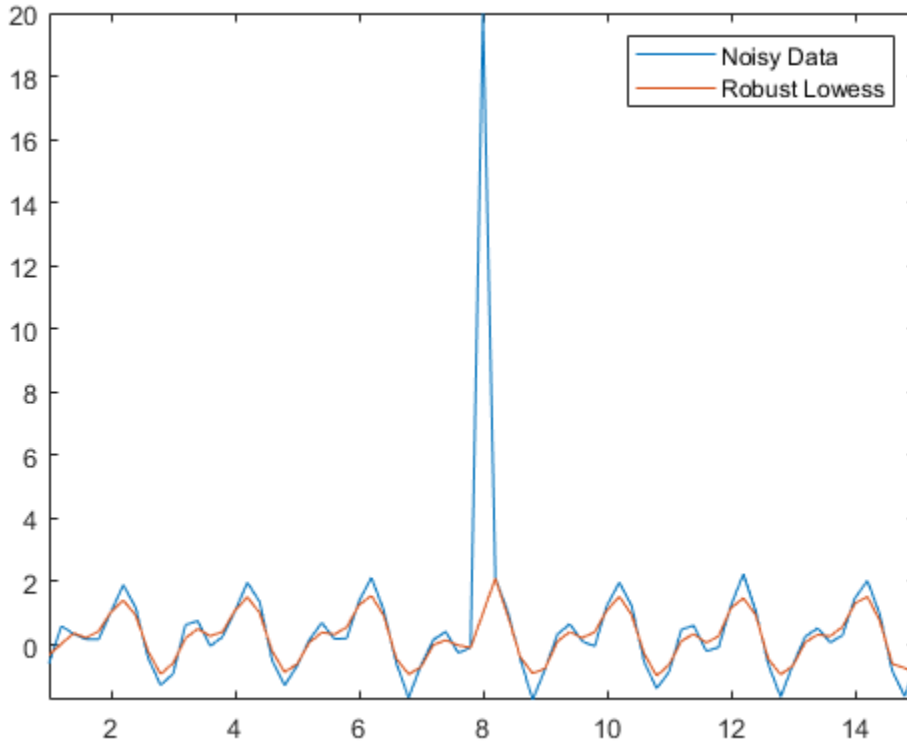


```
window
```

```
window = 3
```

The robust Lowess method is another smoothing method that is particularly helpful when outliers are present in the data in addition to noise. Inject an outlier into the noisy data, and use robust Loess to smooth the data, which eliminates the outlier.

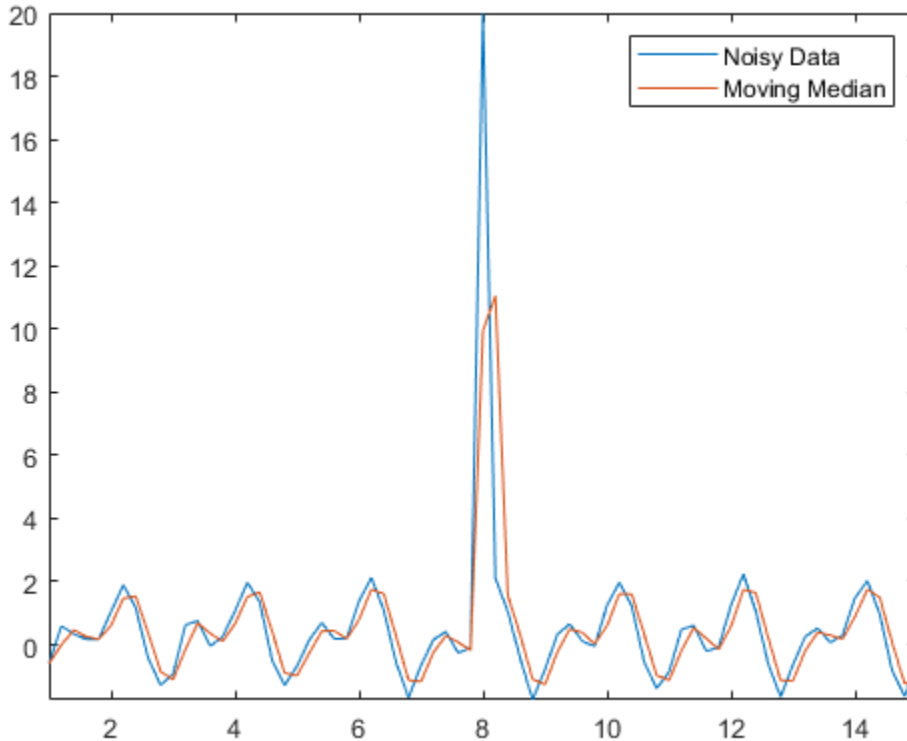
```
Anoise(36) = 20;
Arlowess = smoothdata(Anoise,'rloess',5);
plot(t,Anoise,t,Arlowess)
axis tight
legend('Noisy Data','Robust Lowess')
```



### Detecting Outliers

Outliers in data can significantly skew data processing results and other computed quantities. For example, if you try to smooth data containing outliers with a moving median, you can get misleading peaks or valleys.

```
Amedian = smoothdata(Anoise,'movmedian');  
plot(t,Anoise,t,Amedian)  
axis tight  
legend('Noisy Data','Moving Median')
```



The `isoutlier` function returns a logical 1 when an outlier is detected. Verify the index and value of the outlier in `Anoise`.

```
TF = isoutlier(Anoise);
ind = find(TF)
```

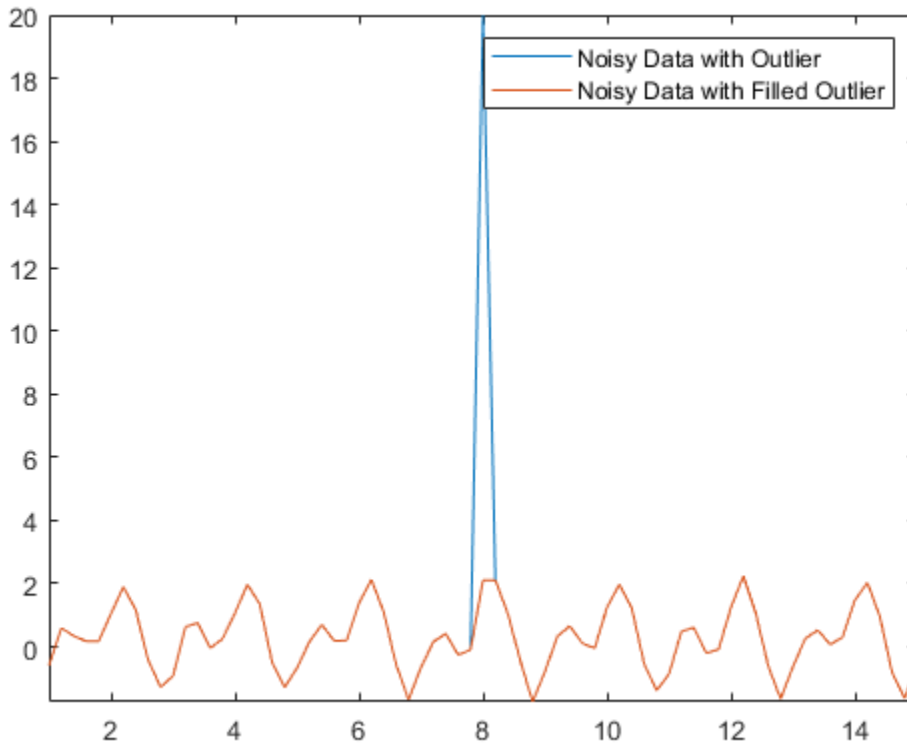
```
ind = 36
```

```
Aoutlier = Anoise(ind)
```

```
Aoutlier = 20
```

You can use the `filloutliers` function to replace outliers in your data by specifying a fill method. For example, fill the outlier in `Anoise` with the value of its neighbor immediately to the right.

```
Afill = filloutliers(Anoise,'next');  
plot(t,Anoise,t,Afill)  
axis tight  
legend('Noisy Data with Outlier','Noisy Data with Filled Outlier')
```



### Nonuniform Data

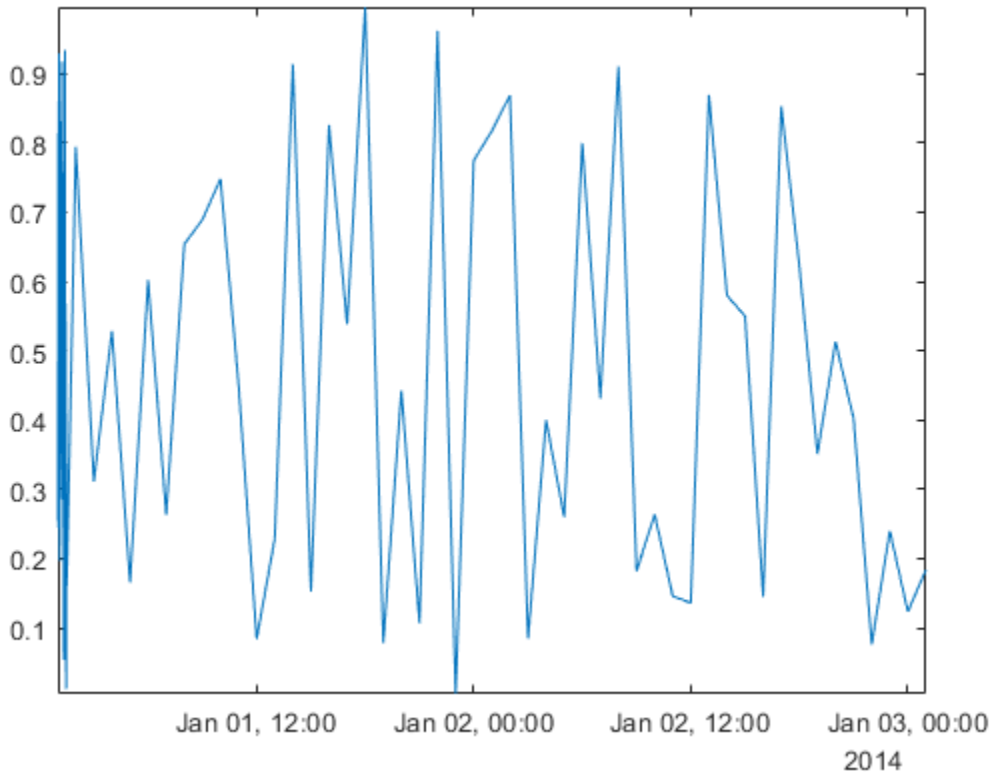
Not all data consists of equally spaced points, which can affect methods for data processing. Create a datetime vector that contains irregular sampling times for the data in Arand. The time vector represents samples taken every minute for the first 30 minutes, then hourly over two days.

```
t0 = datetime(2014,1,1,1,1,1);  
timeminutes = sort(t0 + minutes(1:30));
```

```

timehours = t0 + hours(1:48);
time = [timeminutes timehours];
Airreg = rand(1,length(time));
plot(time,Airreg)
axis tight

```

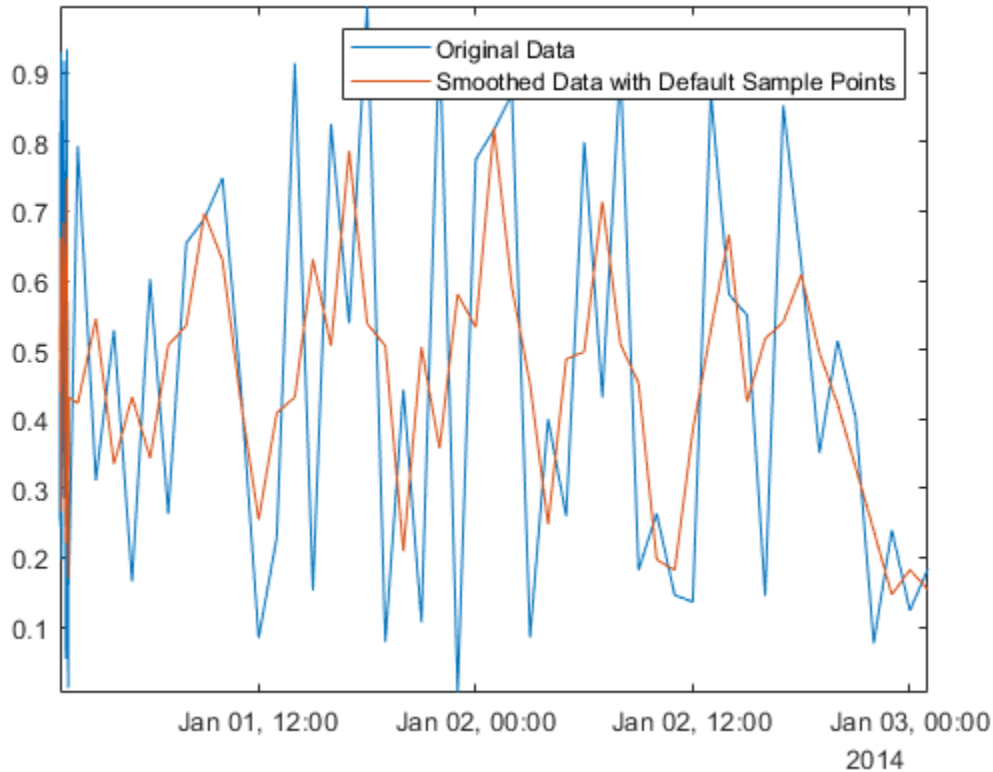


By default, `smoothdata` smooths with respect to equally spaced integers, in this case,  $1, 2, \dots, 78$ . Since integer time stamps do not coordinate with the sampling of the points in `Airreg`, the first half hour of data still appears noisy after smoothing.

```

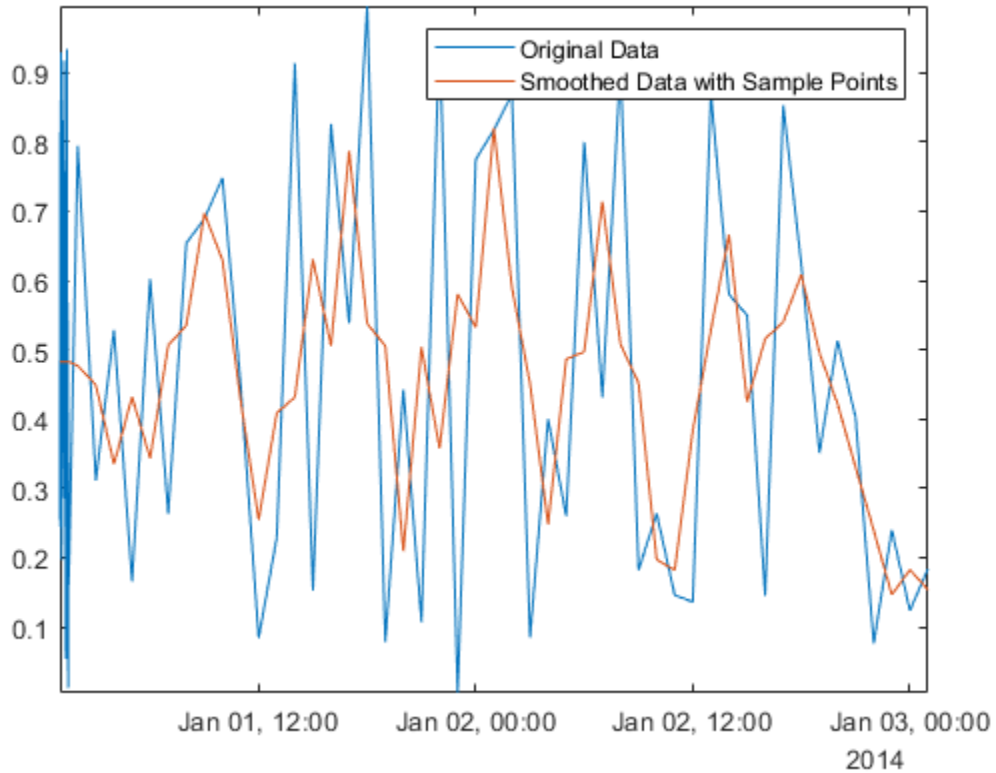
Adefault = smoothdata(Airreg,'movmean',3);
plot(time,Airreg,time,Adefault)
axis tight
legend('Original Data','Smoothed Data with Default Sample Points')

```



Many data processing functions in MATLAB®, including `smoothdata`, `movmean`, and `filloutliers`, allow you to provide sample points, ensuring that data is processed relative to its sampling units and frequencies. To remove the high-frequency variation in the first half hour of data in `Airreg`, use the `'SamplePoints'` option with the time stamps in time.

```
Asamplepoints = smoothdata(Airreg,'movmean',hours(3),'SamplePoints',time);  
plot(time,Airreg,time,Asamplepoints)  
axis tight  
legend('Original Data','Smoothed Data with Sample Points')
```



## See Also

`smoothdata` | `isoutlier` | `filloutliers` | `movmean` | `movmedian`

## Related Examples

- “Filter Data” on page 1-26

## Inconsistent Data

When you examine a data plot, you might find that some points appear to differ dramatically from the rest of the data. In some cases, it is reasonable to consider such points *outliers*, or data values that appear to be inconsistent with the rest of the data.

The following example illustrates how to remove outliers from three data sets in the 24-by-3 matrix `count`. In this case, an outlier is defined as a value that is more than three standard deviations away from the mean.

---

**Caution** Be cautious about changing data unless you are confident that you understand the source of the problem you want to correct. Removing an outlier has a greater effect on the standard deviation than on the mean of the data. Deleting one such point leads to a smaller new standard deviation, which might result in making some remaining points appear to be outliers!

---

```
% Import the sample data
load count.dat;
% Calculate the mean and the standard deviation
% of each data column in the matrix
mu = mean(count)
sigma = std(count)
```

The Command Window displays

```
mu =
    32.0000    46.5417    65.5833

sigma =
    25.3703    41.4057    68.0281
```

When an *outlier* is considered to be more than three standard deviations away from the mean, use the following syntax to determine the number of outliers in each column of the `count` matrix:

```
[n,p] = size(count);
% Create a matrix of mean values by
% replicating the mu vector for n rows
MeanMat = repmat(mu,n,1);
% Create a matrix of standard deviation values by
% replicating the sigma vector for n rows
```



```
SigmaMat = repmat(sigma,n,1);  
% Create a matrix of zeros and ones, where ones indicate  
% the location of outliers  
outliers = abs(count - MeanMat) > 3*SigmaMat;  
% Calculate the number of outliers in each column  
nout = sum(outliers)
```

The procedure returns the following number of outliers in each column:

```
nout =  
      1    0    0
```

There is one outlier in the first data column of `count` and none in the other two columns.

To remove an entire row of data containing the outlier, type

```
count(any(outliers,2),:) = [];
```

Here, `any(outliers,2)` returns a 1 when any of the elements in the `outliers` vector are nonzero. The argument 2 specifies that `any` works down the second dimension of the `count` matrix—its columns.

## Filter Data

### Filter Difference Equation

Filters are data processing techniques that can smooth out high-frequency fluctuations in data or remove periodic trends of a specific frequency from data. In MATLAB, the `filter` function filters a vector of data  $x$  according to the following difference equation, which describes a tapped delay-line filter.

$$a(1)y(n) = b(1)x(n) + b(2)x(n-1) + \dots + b(N_b)x(n-N_b+1) \\ - a(2)y(n-1) - \dots - a(N_a)y(n-N_a+1)$$

In this equation,  $a$  and  $b$  are vectors of coefficients of the filter,  $N_a$  is the feedback filter order, and  $N_b$  is the feedforward filter order.  $n$  is the index of the current element of  $x$ . The output  $y(n)$  is a linear combination of the current and previous elements of  $x$  and  $y$ .

The `filter` function uses specified coefficient vectors  $a$  and  $b$  to filter the input data  $x$ . For more information on difference equations describing filters, see [1].

### Moving-Average Filter of Traffic Data

The `filter` function is one way to implement a moving-average filter, which is a common data smoothing technique.

The following difference equation describes a filter that averages time-dependent data with respect to the current hour and the three previous hours of data.

$$y(n) = \frac{1}{4}x(n) + \frac{1}{4}x(n-1) + \frac{1}{4}x(n-2) + \frac{1}{4}x(n-3)$$

Import data that describes traffic flow over time, and assign the first column of vehicle counts to the vector  $x$ .

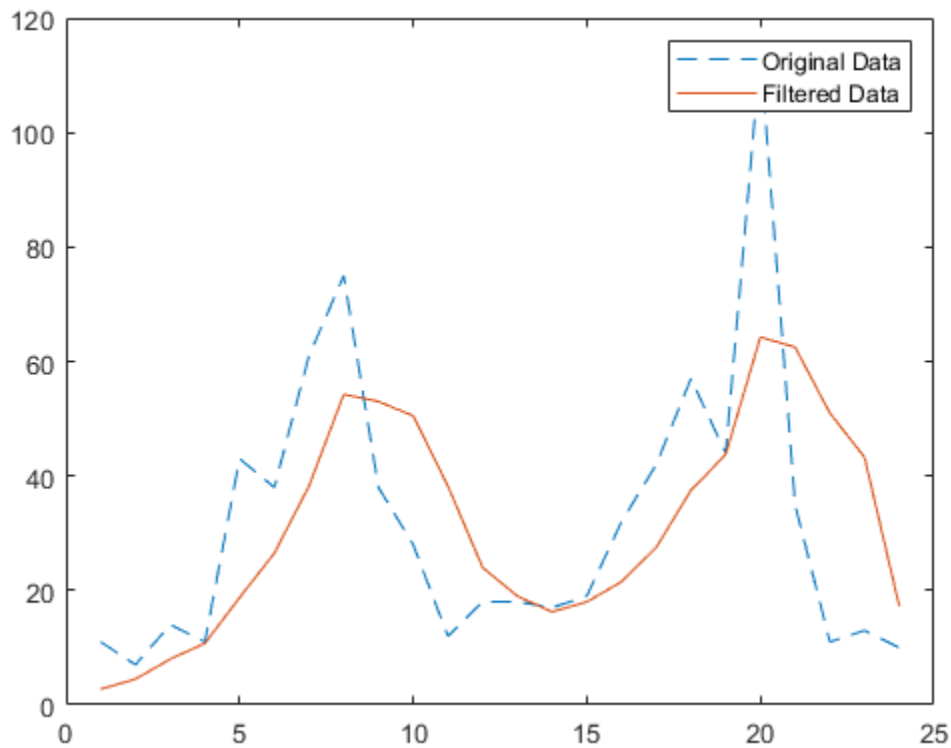
```
load count.dat  
x = count(:,1);
```

Create the filter coefficient vectors.

```
a = 1;  
b = [1/4 1/4 1/4 1/4];
```

Compute the 4-hour moving average of the data, and plot both the original data and the filtered data.

```
y = filter(b,a,x);  
  
t = 1:length(x);  
plot(t,x,'--',t,y,'-')  
legend('Original Data','Filtered Data')
```



## Modify Amplitude of Data

This example shows how to modify the amplitude of a vector of data by applying a transfer function.

In digital signal processing, filters are often represented by a transfer function. The Z-transform of the difference equation

$$a(1)y(n) = b(1)x(n) + b(2)x(n-1) + \dots + b(N_b)x(n-N_b+1) \\ - a(2)y(n-1) - \dots - a(N_a)y(n-N_a+1)$$

is the following transfer function.

$$Y(z) = H(z^{-1})X(z) = \frac{b(1) + b(2)z^{-1} + \dots + b(N_b)z^{-N_b+1}}{a(1) + a(2)z^{-1} + \dots + a(N_a)z^{-N_a+1}} X(z)$$

Use the transfer function

$$H(z^{-1}) = \frac{b(z^{-1})}{a(z^{-1})} = \frac{2 + 3z^{-1}}{1 + 0.2z^{-1}}$$

to modify the amplitude of the data in `count.dat`.

Load the data and assign the first column to the vector `x`.

```
load count.dat
x = count(:,1);
```

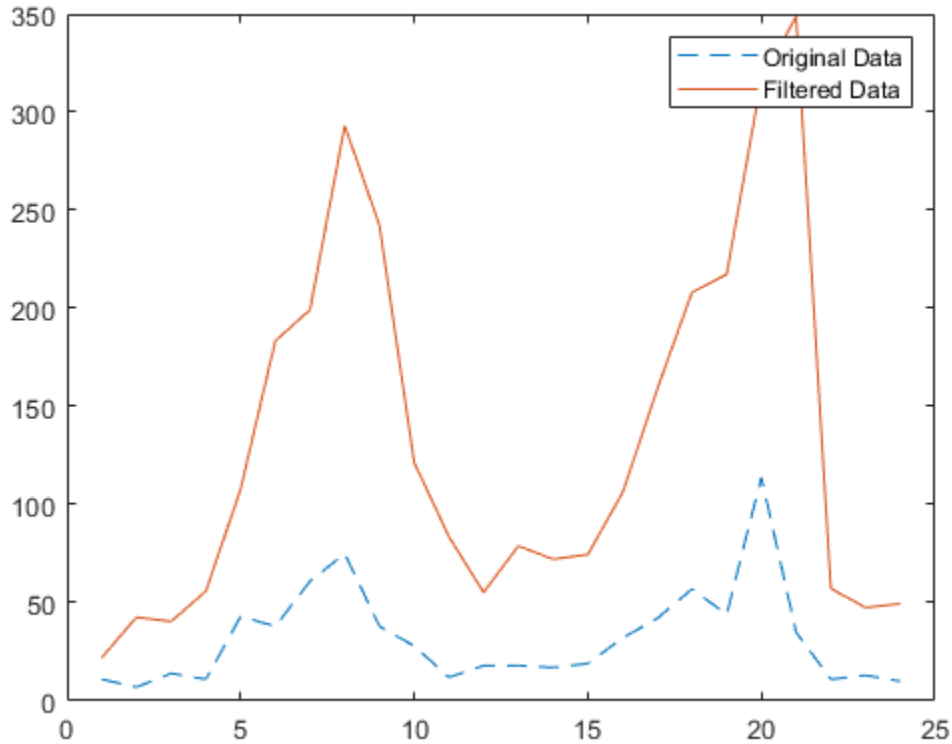
Create the filter coefficient vectors according to the transfer function  $H(z^{-1})$ .

```
a = [1 0.2];
b = [2 3];
```

Compute the filtered data, and plot both the original data and the filtered data. This filter primarily modifies the amplitude of the original data.

```
y = filter(b,a,x);

t = 1:length(x);
plot(t,x,'--',t,y,'-')
legend('Original Data','Filtered Data')
```



## References

- [1] Oppenheim, Alan V., Ronald W. Schafer, and John R. Buck. *Discrete-Time Signal Processing*. Upper Saddle River, NJ: Prentice-Hall, 1999.

## See Also

`conv` | `filter` | `filter2` | `movmean` | `smoothdata`

## **Related Examples**

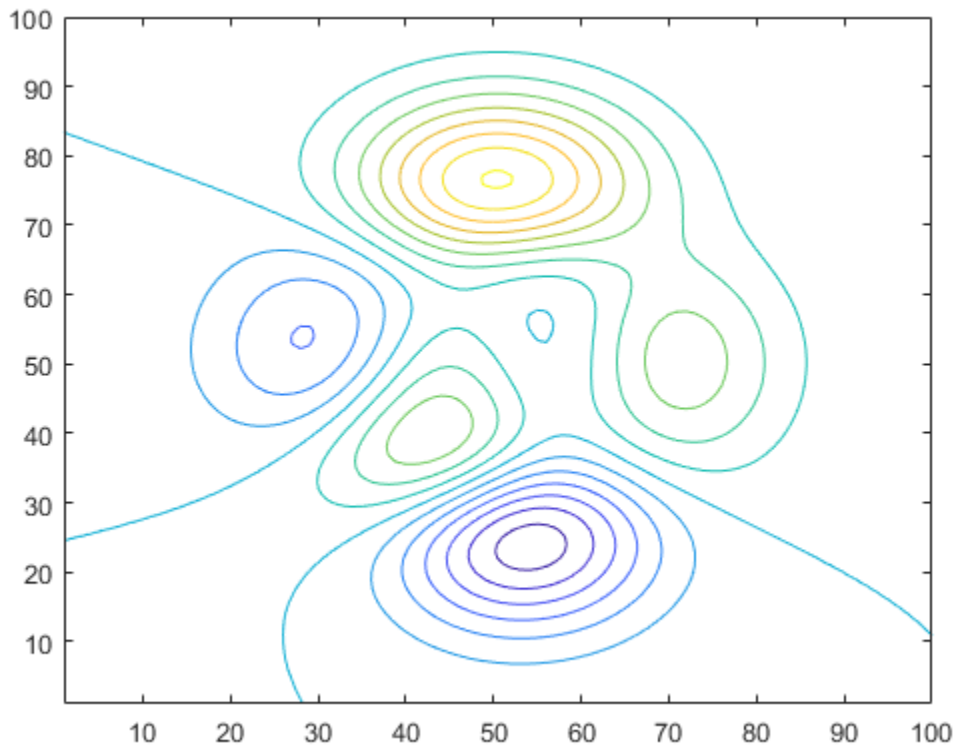
- “Smooth Data with Convolution” on page 1-31

## Smooth Data with Convolution

You can use convolution to smooth 2-D data that contains high-frequency components.

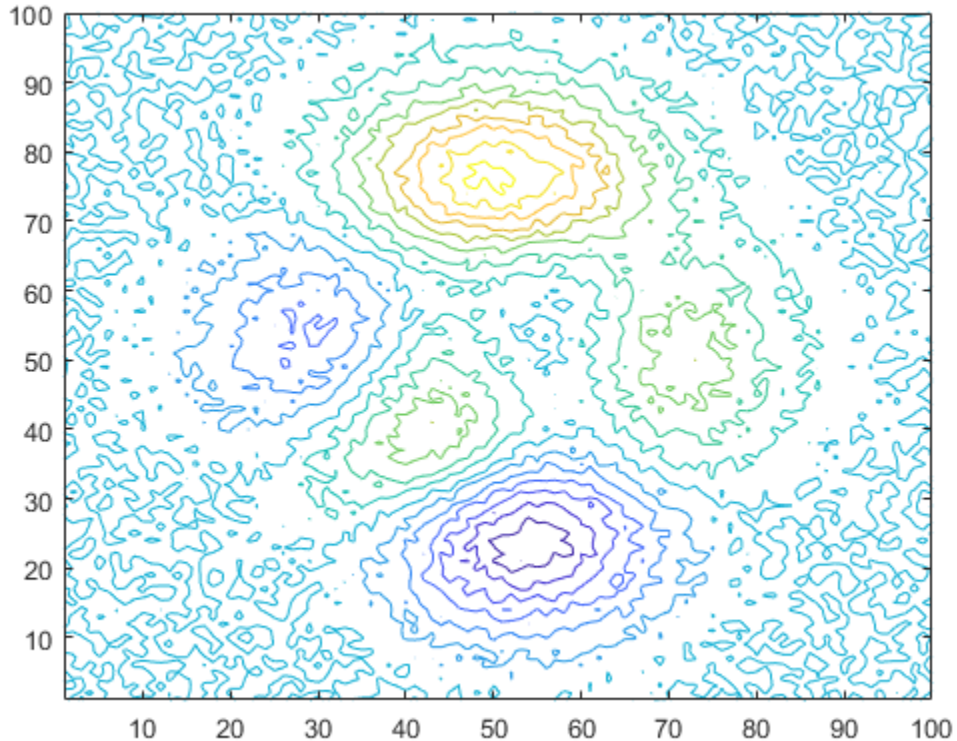
Create 2-D data using the peaks function, and plot the data at various contour levels.

```
Z = peaks(100);  
levels = -7:1:10;  
contour(Z,levels)
```



Inject random noise into the data and plot the noisy contours.

```
Znoise = Z + rand(100) - 0.5;  
contour(Znoise,levels)
```

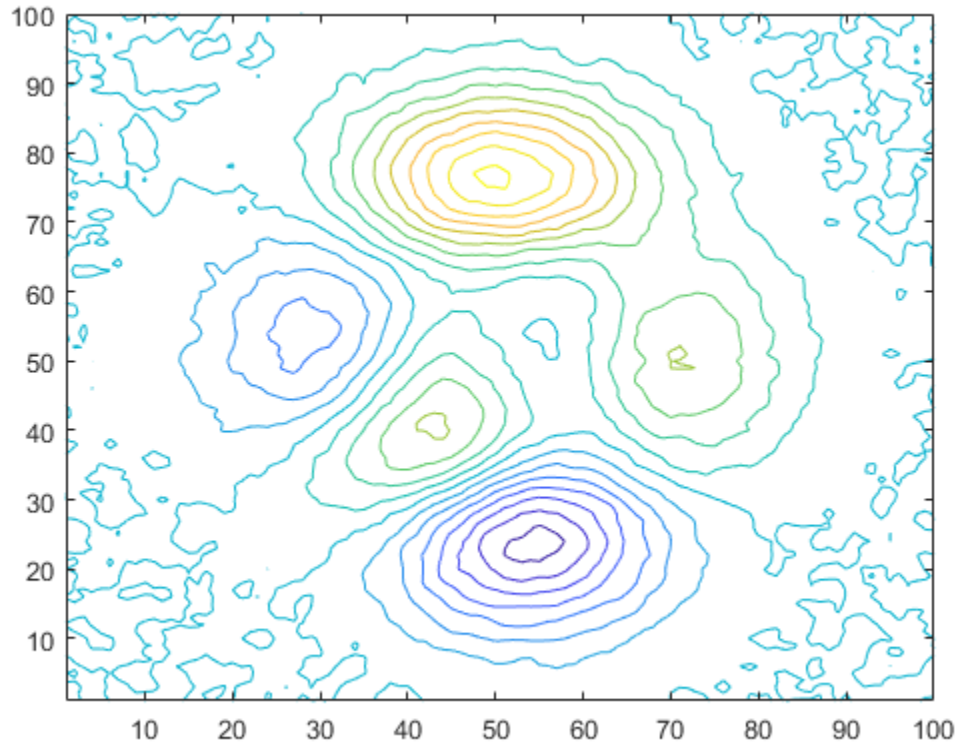


The `conv2` function in MATLAB® convolves 2-D data with a specified kernel whose elements define how to remove or enhance features of the original data. Kernels do not have to be the same size as the input data. Small-sized kernels can be sufficient to smooth data containing only a few frequency components. Larger sized kernels can provide more precision for tuning frequency response, resulting in smoother output.

Define a 3-by-3 kernel `K` and use `conv2` to smooth the noisy data in `Znoise`. Plot the smoothed contours. The `'same'` option in `conv2` makes the output the same size as the input.

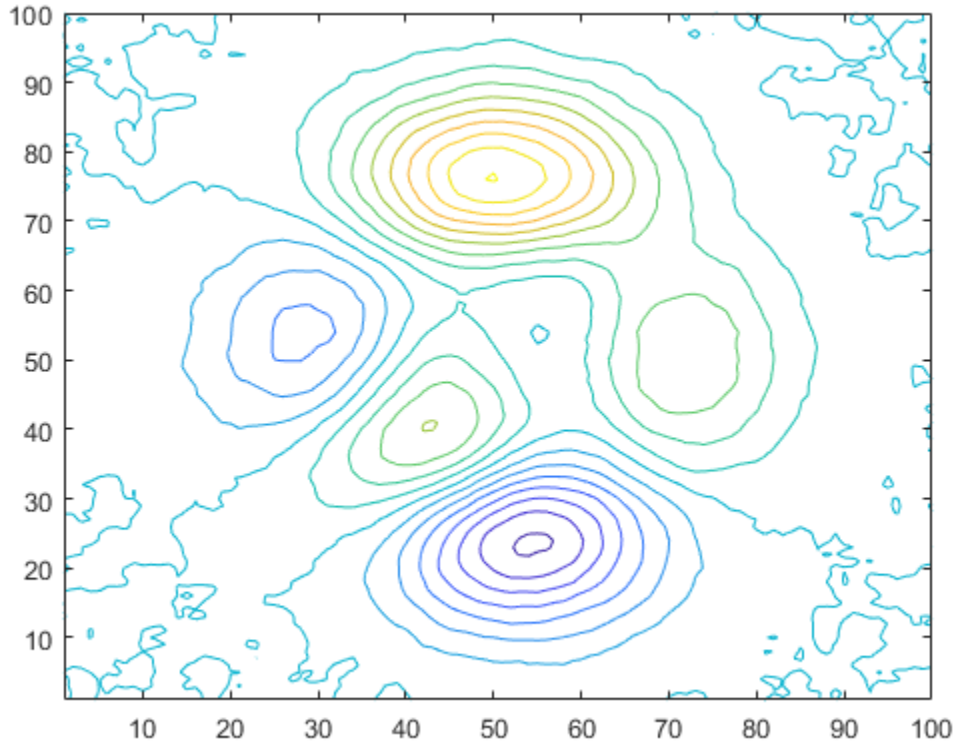
```
K = 0.125*ones(3);  
Zsmooth1 = conv2(Znoise,K,'same');  
contour(Zsmooth1, levels)
```





Smooth the noisy data with a 5-by-5 kernel, and plot the new contours.

```
K = 0.045*ones(5);  
Zsmooth2 = conv2(Znoise,K, 'same');  
contour(Zsmooth2,levels)
```



## See Also

`conv` | `conv2` | `filter` | `smoothdata`

## Related Examples

- “Filter Data” on page 1-26

# Detrending Data

**In this section...**

“Introduction” on page 1-35

“Remove Linear Trends from Data” on page 1-35

## Introduction

The MATLAB function `detrend` subtracts the mean or a best-fit line (in the least-squares sense) from your data. If your data contains several data columns, `detrend` treats each data column separately.

Removing a trend from the data enables you to focus your analysis on the fluctuations in the data about the trend. A linear trend typically indicates a systematic increase or decrease in the data. A systematic shift can result from sensor drift, for example. While trends can be meaningful, some types of analyses yield better insight once you remove trends.

Whether it makes sense to remove trend effects in the data often depends on the objectives of your analysis.

## Remove Linear Trends from Data

This example shows how to remove a linear trend from daily closing stock prices to emphasize the price fluctuations about the overall increase. If the data does have a trend, detrending it forces its mean to zero and reduces overall variation. The example simulates stock price fluctuations using a distribution taken from the `gallery` function.

Create a simulated data set and compute its mean. `sdata` represents the daily price changes of a stock.

```
t = 0:300;  
dailyFluct = gallery('normaldata',size(t),2);  
sdata = cumsum(dailyFluct) + 20 + t/100;
```

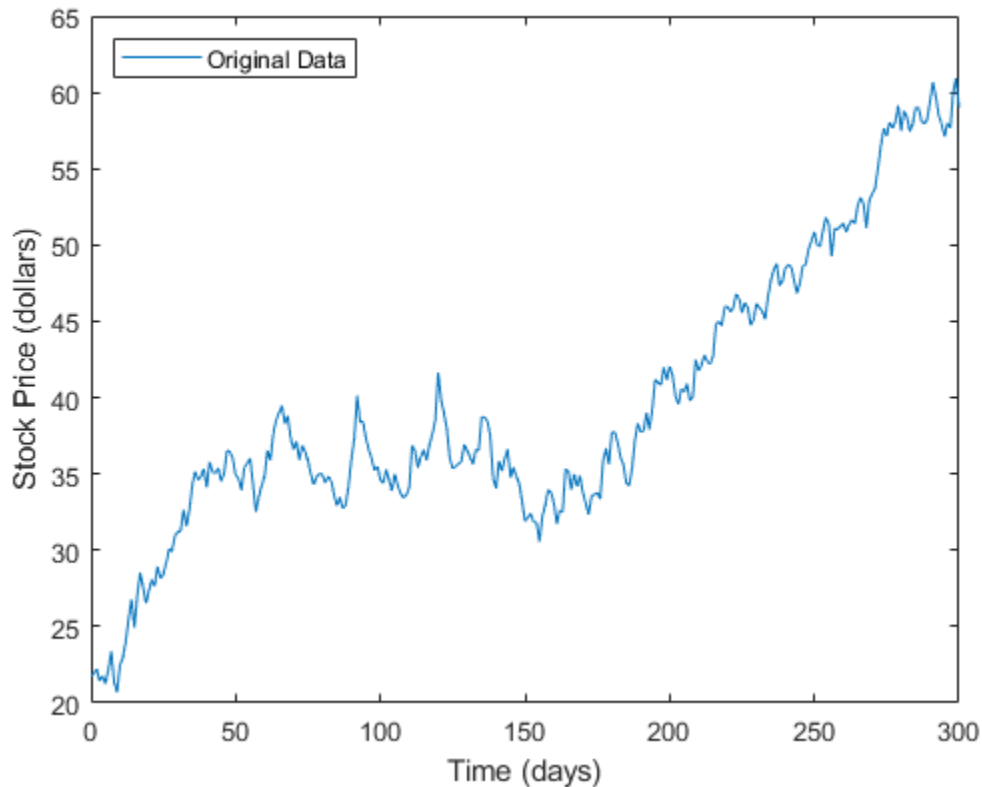
Find the average of the data.

```
mean(sdata)
```

```
ans = 39.4851
```

Plot and label the data. Notice the systematic increase in the stock prices that the data displays.

```
figure
plot(t,sdata);
legend('Original Data','Location','northwest');
xlabel('Time (days)');
ylabel('Stock Price (dollars)');
```



Apply `detrend`, which performs a linear fit to `sdata` and then removes the trend from it. Subtracting the output from the input yields the computed trend line.

```
detrend_sdata = detrend(sdata);
trend = sdata - detrend_sdata;
```

Find the average of the detrended data.

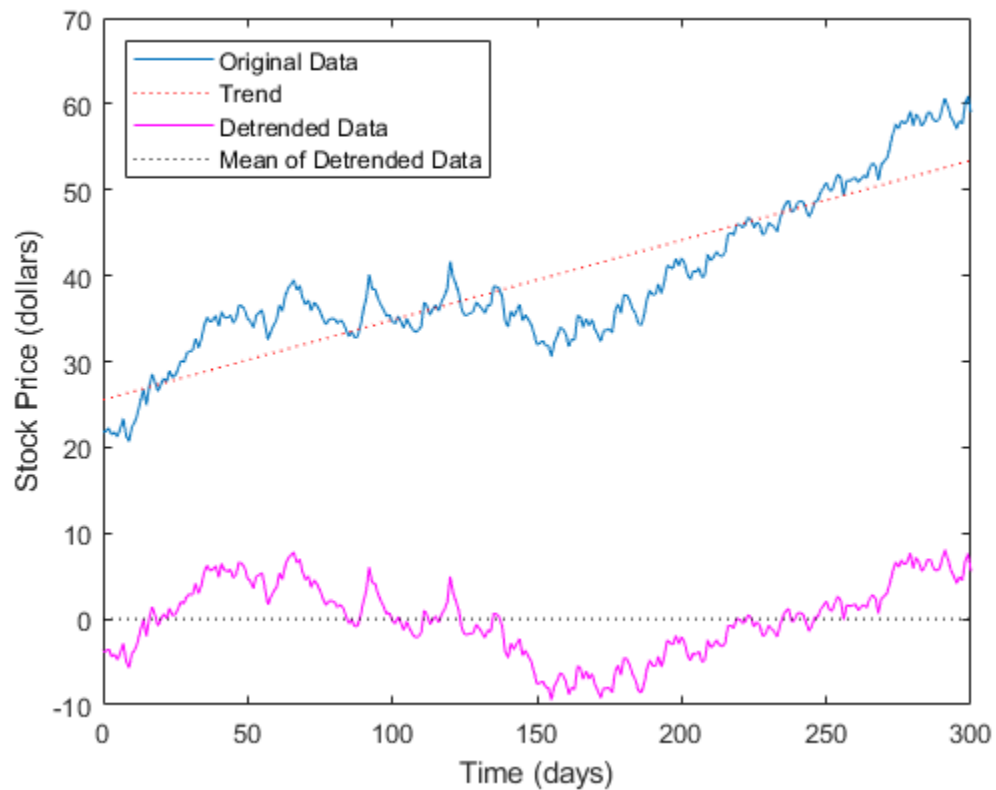
```
mean(detrend_sdata)
```

```
ans = 1.1425e-14
```

As expected, the detrended data has a mean very close to 0.

Display the results by adding the trend line, the detrended data, and its mean to the graph.

```
hold on
plot(t,trend,':r')
plot(t,detrend_sdata,'m')
plot(t,zeros(size(t)),':k')
legend('Original Data','Trend','Detrended Data',...
       'Mean of Detrended Data','Location','northwest')
xlabel('Time (days)');
ylabel('Stock Price (dollars)');
```



## See Also

[cumsum](#) | [detrrend](#) | [gallery](#) | [plot](#)

## Descriptive Statistics

### In this section...

“Functions for Calculating Descriptive Statistics” on page 1-39

“Example: Using MATLAB Data Statistics” on page 1-41

If you need more advanced statistics features, you might want to use the Statistics and Machine Learning Toolbox™ software.

### Functions for Calculating Descriptive Statistics

Use the following MATLAB functions to calculate the descriptive statistics for your data.

**Note** For matrix data, descriptive statistics for each column are calculated independently.

#### Statistics Function Summary

Function	Description
max	Maximum value
mean	Average or mean value
median	Median value
min	Smallest value
mode	Most frequent value
std	Standard deviation
var	Variance, which measures the spread or dispersion of the values

The following examples apply MATLAB functions to calculate descriptive statistics:

- “Example 1 — Calculating Maximum, Mean, and Standard Deviation” on page 1-40
- “Example 2 — Subtracting the Mean” on page 1-41

**Example 1 — Calculating Maximum, Mean, and Standard Deviation**

This example shows how to use MATLAB functions to calculate the maximum, mean, and standard deviation values for a 24-by-3 matrix called `count`. MATLAB computes these statistics independently for each column in the matrix.

```
% Load the sample data
load count.dat
% Find the maximum value in each column
mx = max(count)
% Calculate the mean of each column
mu = mean(count)
% Calculate the standard deviation of each column
sigma = std(count)
```

The results are

```
mx =
      114      145      257

mu =
  32.0000   46.5417   65.5833

sigma =
  25.3703   41.4057   68.0281
```

To get the row numbers where the maximum data values occur in each data column, specify a second output parameter `indx` to return the row index. For example:

```
[mx,indx] = max(count)
```

These results are

```
mx =
      114      145      257

indx =
      20      20      20
```

Here, the variable `mx` is a row vector that contains the maximum value in each of the three data columns. The variable `indx` contains the row indices in each column that correspond to the maximum values.



To find the minimum value in the entire `count` matrix, 24-by-3 matrix into a 72-by-1 column vector by using the syntax `count(:)`. Then, to find the minimum value in the single column, use the following syntax:

```
min(count(:))  
  
ans =  
    7
```

### Example 2 – Subtracting the Mean

Subtract the mean from each column of the matrix by using the following syntax:

```
% Get the size of the count matrix  
[n,p] = size(count)  
% Compute the mean of each column  
mu = mean(count)  
% Create a matrix of mean values by  
% replicating the mu vector for n rows  
MeanMat = repmat(mu,n,1)  
% Subtract the column mean from each element  
% in that column  
x = count - MeanMat
```

---

**Note** Subtracting the mean from the data is also called *detrending*. For more information about removing the mean or the best-fit line from the data, see “Detrending Data” on page 1-35.

---

### Example: Using MATLAB Data Statistics

The Data Statistics dialog box helps you calculate and plot descriptive statistics with the data. This example shows how to use MATLAB Data Statistics to calculate and plot statistics for a 24-by-3 matrix, called `count`. The data represents how many vehicles passed by traffic counting stations on three streets.

This section contains the following topics:

- “Calculating and Plotting Descriptive Statistics” on page 1-42
- “Formatting Data Statistics on Plots” on page 1-44
- “Saving Statistics to the MATLAB Workspace” on page 1-46

- “Generating Code Files” on page 1-47

---

**Note** MATLAB Data Statistics is available for 2-D plots only.

---

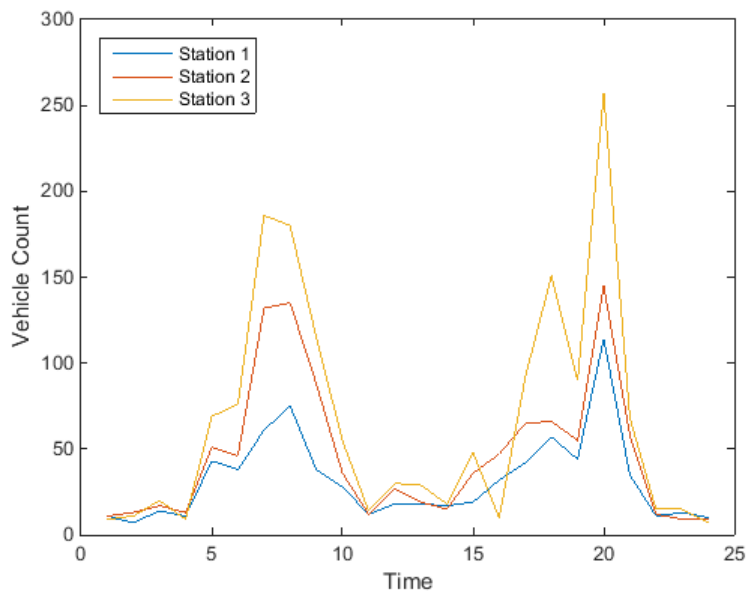
## Calculating and Plotting Descriptive Statistics

- 1 Load and plot the data:

```
load count.dat
[n,p] = size(count);

% Define the x-values
t = 1:n;

% Plot the data and annotate the graph
plot(t,count)
legend('Station 1','Station 2','Station 3','Location','northwest')
xlabel('Time')
ylabel('Vehicle Count')
```



---

**Note** The legend contains the name of each data set, as specified by the `legend` function: `Station 1`, `Station 2`, and `Station 3`. A *data set* refers to each column of data in the array you plotted. If you do not name the data sets, default names are assigned: `data1`, `data2`, and so on.

---

- 2 In the Figure window, select **Tools > Data Statistics**.

The Data Statistics dialog box opens and displays descriptive statistics for the X- and Y-data of the `Station 1` data set.

---

**Note** The Data Statistics dialog box displays a *range*, which is the difference between the minimum and maximum values in the selected data set. The dialog box does not display the range on the plot.

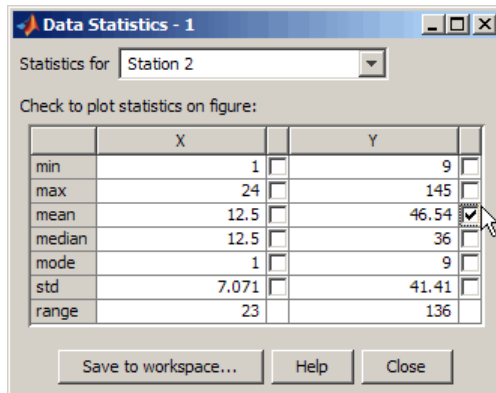
---

- 3 Select a different data set in the **Statistics for** list: `Station 2`.

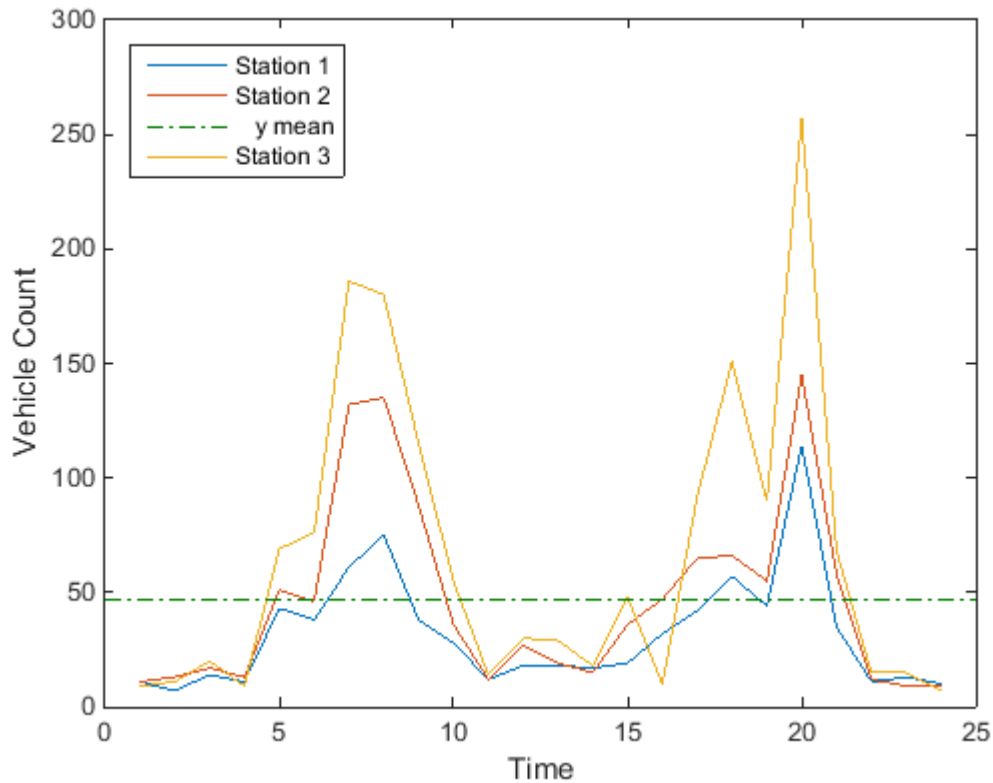
This displays the statistics for the X and Y data of the `Station 2` data set.

- 4 Select the check box for each statistic you want to display on the plot, and then click **Save to workspace**.

For example, to plot the mean of `Station 2`, select the **mean** check box in the **Y** column.



This plots a horizontal line to represent the mean of `Station 2` and updates the legend to include this statistic.



### Formatting Data Statistics on Plots


The Data Statistics dialog box uses colors and line styles to distinguish statistics from the data on the plot. This portion of the example shows how to customize the display of descriptive statistics on a plot, such as the color, line width, line style, or marker.

---

**Note** Do not edit display properties of statistics until you finish plotting all the statistics with the data. If you add or remove statistics after editing plot properties, the changes to plot properties are lost.

---

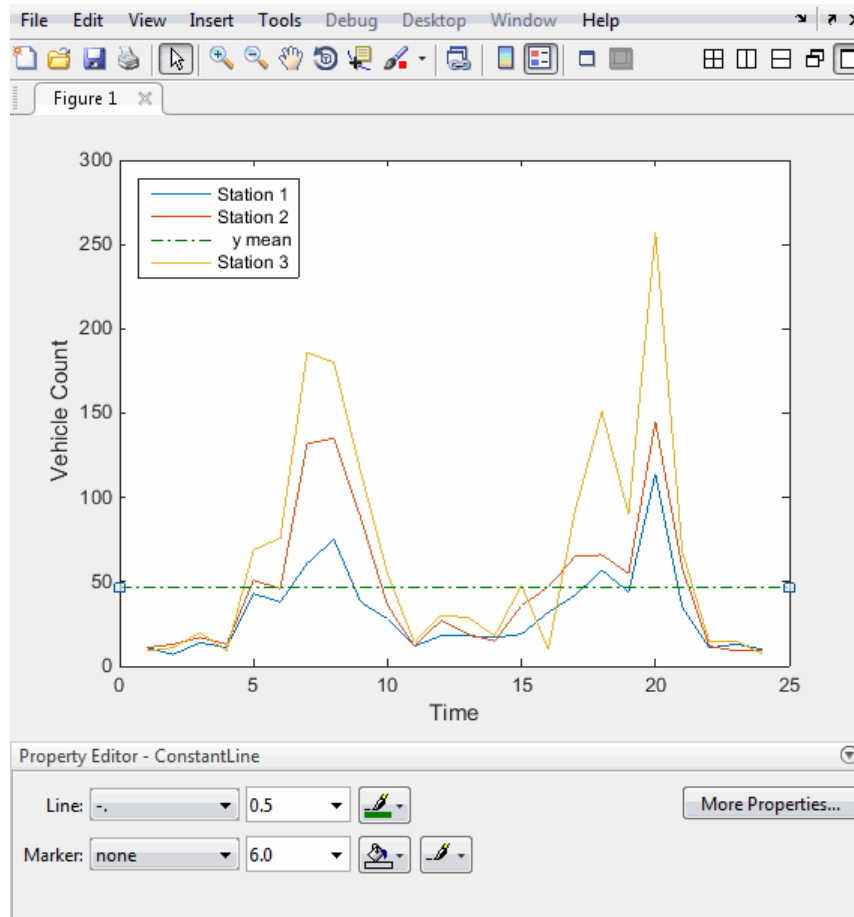
To modify the display of data statistics on a plot:

- 1 In the MATLAB Figure window, click the  (**Edit Plot**) button in the toolbar.

This step enables plot editing.

- 2 Double-click the statistic on the plot for which you want to edit display properties. For example, double-click the horizontal line representing the mean of Station 2.

This step opens the Property Editor below the MATLAB Figure window, where you can modify the appearance of the line used to represent this statistic.



- 3 In the Property Editor, specify the **Line** and **Marker** styles, sizes, and colors.

**Tip** Alternatively, right-click the statistic on the plot, and select an option from the shortcut menu.

---

## Saving Statistics to the MATLAB Workspace

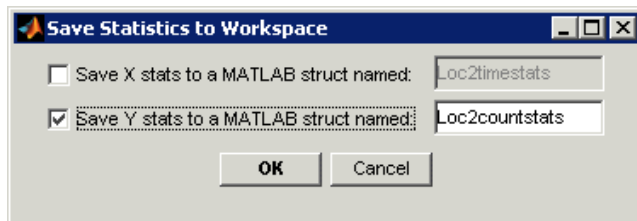
Perform these steps to save the statistics to the MATLAB workspace.

**Note** When your plot contains multiple data sets, save statistics for each data set individually. To display statistics for a different data set, select it from the **Statistics for** list in the Data Statistics dialog box.

---

- 1 In the Data Statistics dialog box, click the **Save to workspace** button.
- 2 In the Save Statistics to Workspace dialog box, select options to save statistics for either X data, Y data, or both. Then, enter the corresponding variable names.

In this example, save only the Y data. Enter the variable name as `Loc2countstats`.



- 3 Click **OK**.

This step saves the descriptive statistics to a structure. The new variable is added to the MATLAB workspace.

To view the new structure variable, type the variable name at the MATLAB prompt:

```
Loc2countstats
```

```
Loc2countstats =
```

```
    min: 9  
    max: 145  
   mean: 46.5417  
  median: 36  
    mode: 9
```

```
std: 41.4057  
range: 136
```

### Generating Code Files

This portion of the example shows how to generate a file containing MATLAB code that reproduces the format of the plot and the plotted statistics with new data.

- 1 In the Figure window, select **File > Generate Code**.

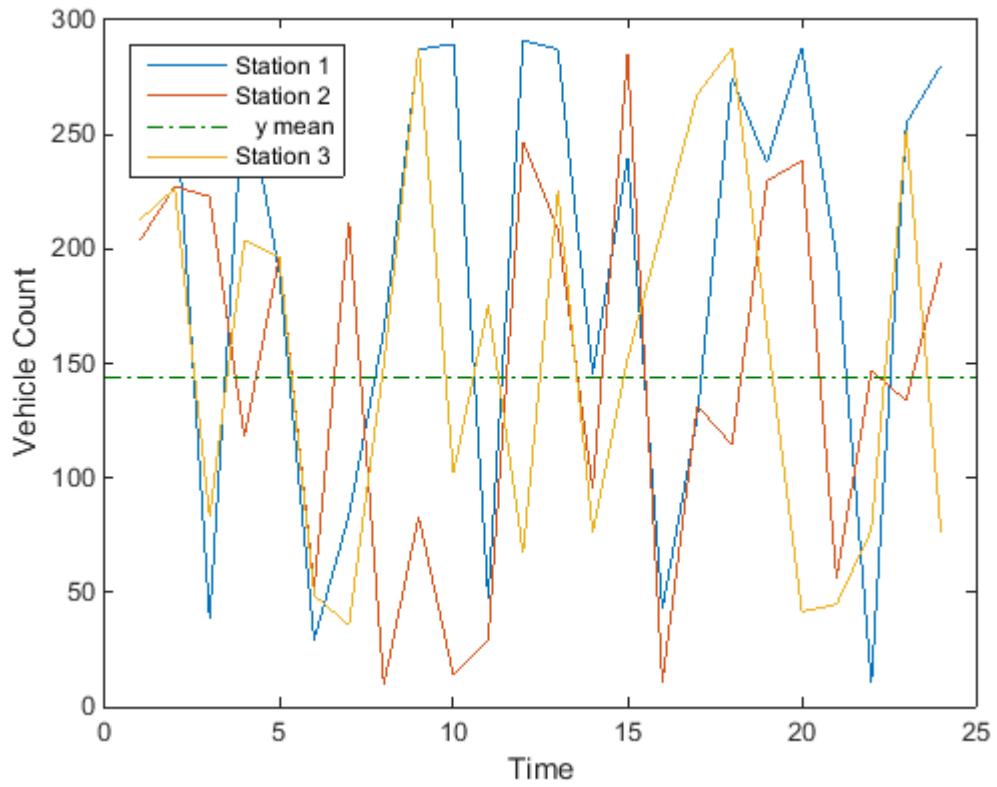
This step creates a function code file and displays it in the MATLAB Editor.

- 2 Change the name of the function on the first line of the file from `createfigure` to something more specific, like `countplot`. Save the file to your current folder with the file name `countplot.m`.
- 3 Generate some new, random count data:

```
randcount = 300*rand(24,3);
```

- 4 Reproduce the plot with the new data and the recomputed statistics:

```
countplot(t, randcount)
```





# Interactive Data Exploration

---

- “What Is Interactive Data Exploration?” on page 2-2
- “Marking Up Graphs with Data Brushing” on page 2-4
- “Making Graphs Responsive with Data Linking” on page 2-12
- “Interacting with Graphed Data” on page 2-22

# What Is Interactive Data Exploration?

## Interacting with MATLAB Data Graphs

The MATLAB data analysis and graphics tools for visual data exploration include the following capabilities:

- Highlighting and editing observations on graphs with data brushing on page 2-4
- Connecting data graphs with variables with data linking on page 2-12
- Finding, adding, removing, and changing data values with the “Data Brushing with the Variables Editor” on page 2-22
- Describing observations on graphs with data tips on page 2-23

Used alone or together, these tools help you to perceive trends, noise, and relationships in data sets, and understand aspects of the phenomena you model.

## Understanding Data Using Graphic Presentations

Finding patterns in numbers is a mathematical and an intuitive undertaking. When people collect data to analyze, they often want to see how models, variables, and constants explain hypotheses. Sometimes they see patterns by scanning tables or sets of statistics, other times by contemplating graphical representations of models and data. An analyst's powers of pattern recognition can lead to insights into data's distribution, outliers, curvilinearity, associations between variables, goodness-of-fit to models, and more. Computers amplify those powers greatly.

Graphically exploring digital data interactively generally requires:

- Data displays for charts, graphs, and maps
- A graphical user interface (UI) capable of directly manipulating the displays
- Software that categorizes selected data performs operations on the categories, and then updates or creates new data displays

This approach to understanding is often called exploratory data analysis (EDA), a term coined during the infancy of computer graphics in the 1970s and generally attributed to statistician John Tukey (who also invented the box plot). EDA complements statistical methods and tools to help analysts check hypotheses and validate models. An EDA UI usually lets analysts divide observations of variables on data plots into subsets using mouse gestures, and then analyze further or eliminate selected observations.

Part of EDA is simply looking at data graphics with an informed eye to observe patterns or lack of them. What makes EDA especially powerful, however, are interactive tools that let analysts probe, drill down, map, and spin data sets around, and select observations and trace them through plots, tables, and models.

Well before digital tool sets like the MATLAB environment developed, curious quantitative types plotted graphs, maps, and other data diagrams to trigger insights into what their collections of numbers might mean. If you are curious about what data might mean and like to reflect on data graphics, MATLAB provides many options:

- Plotting data — scatter, line, area, bar, histogram and other types of graphs
- Plotting thematic maps to show spatial relationships of point, lines and area data
- Plotting N-D point, vector, contour, surface, and volume shapes
- Overlaying other variables on points, lines, and surfaces (e.g. texture-maps)
- Rendering portions of a 3-D display with transparency
- Animating any of the above

All of these options generate static or dynamic displays that may reveal meaning in data. In many environments, however, users cannot interact with them; they can only change data or parameters and redisplay the same or different data graphics. MATLAB tools enable users to directly manipulate data displays to explore correlations and anomalies in data sets, as the following sections explain.

## Marking Up Graphs with Data Brushing

### In this section...

“What Is Data Brushing?” on page 2-4

“How to Brush Data” on page 2-5

“Effects of Brushing on Data” on page 2-8


“Other Data Brushing Aspects” on page 2-10

### What Is Data Brushing?


When you brush data, you manually select observations on an interactive data display in the course of assessing validity, testing hypotheses, or segregating observations for further processing. You can brush data on 2-D graphs, 3-D graphs, and surfaces. Most of the MATLAB high-level plotting functions allow you to brush on their displays. For a list of these functions, see “Types of Charts You Can Brush” in the `brush` function reference page.

Data brushing is a MATLAB figure interactive mode like zooming, panning or plot editing. You can use data brushing mode to select, remove, and replace individual data values.

Activate data brushing in any of these ways:

- Click  on the figure toolbar.
- Select **Tools > Brush**.
- Right-click a cell in the Variables editor and select **Brushing > Brushing on**.
- Call the `brush` function.

The figure toolbar data brushing button contains two parts:


- Data brushing button  that toggles data brushing on and off.
- Data brushing button arrow ▼ that displays a drop-down menu for choosing the brushing color.

You also can set the color with the `brush` function; it accepts `ColorSpec` names and RGB triplets. For example:

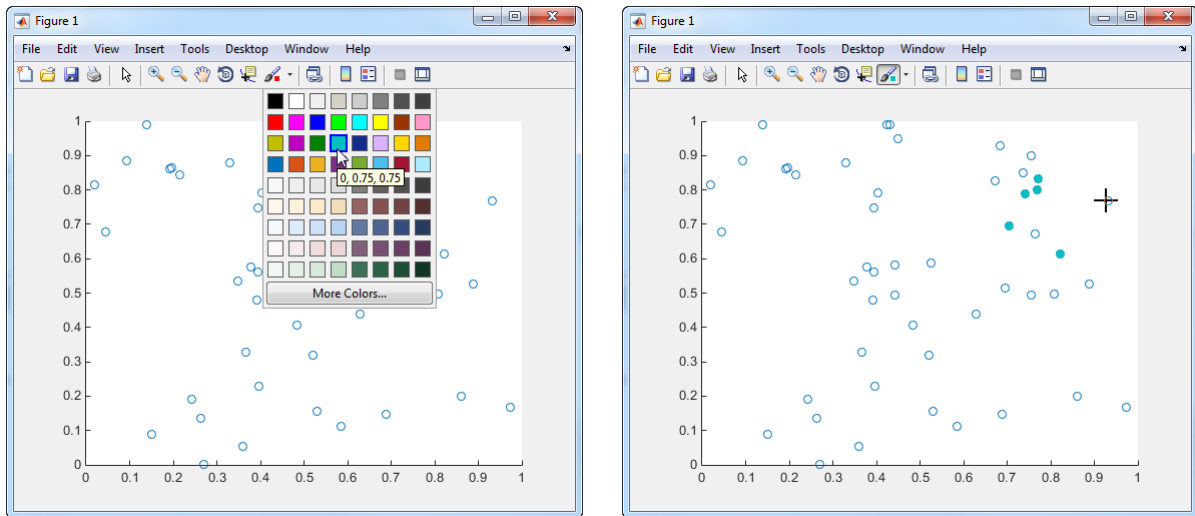
```
brush magenta  
brush([.1 .3 .5])
```

## How to Brush Data

To brush observations on graphs and surface plots,

- 1** To enter brushing mode, select the Data Brushing button  in the figure toolbar. You also can select a brushing color with the Data Brushing button arrow ▼.
- 2** Drag a selection rectangle to highlight observations on a graph in the current brushing color.  
Instead of dragging out a rectangle, you can click any observation to select it. Double-clicking selects all the observations in a series.
- 3** To add other observations to the highlighted set, hold down the **Shift** key and brush them.
- 4** **Shift**+clicking or **Shift**+dragging highlighted observations eliminates their highlighting and removes them from the selection set; this lets you select any set of observations.

The following figures show a scatter plot before and after brushing some outlying observations; the left-hand plot displays the Data Brushing tool palette for choosing a brush color.



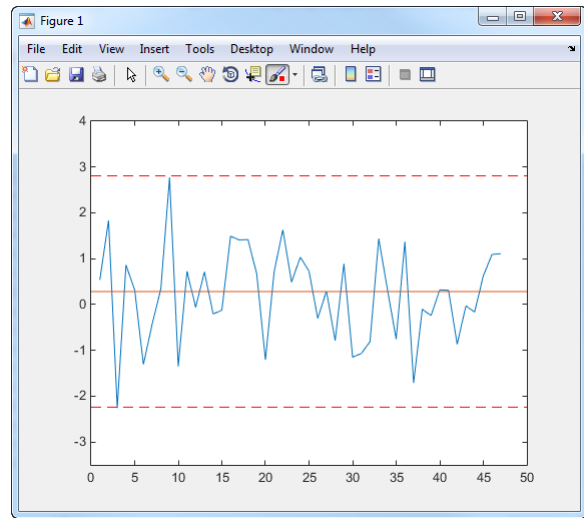
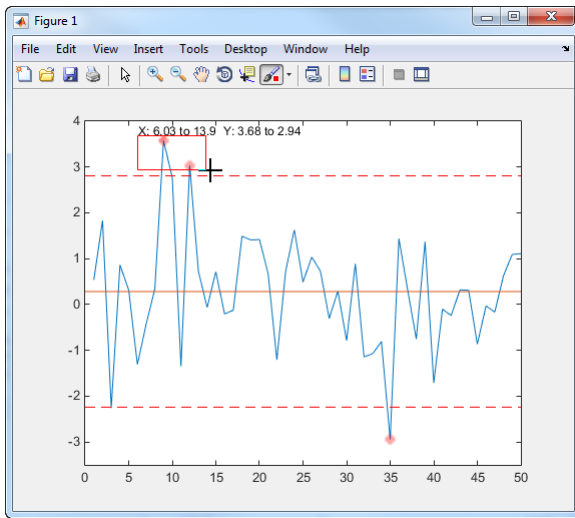
Brushed observations remain brushed even in other modes (pan, zoom, edit) until you deselect them by brushing an empty area or by selecting **Clear all brushing** from the context menu. You can add and remove data tips to a brushed plot without disturbing its brushing.

Once you have brushed observations from one or more graphed variables, you can perform several tasks with the brushing set, either from the **Tools** menu or by right-clicking any brushed observation:

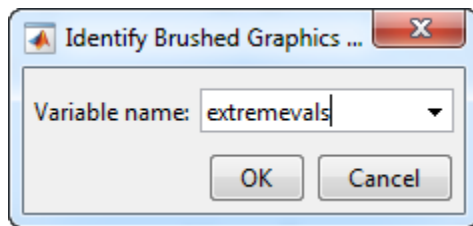
- Remove all brushed observations from the plot.
- Remove all unbrushed observations from the plot.
- Replace the brushed observations with NaN or constant values.
- Copy the brushed data values to the clipboard.
- Paste the brushed data values to the command window
- Create a variable to hold the brushed data values
- Clear brushing marks from the plot (context menu only)

The two following figures show a lineseries plot of a variable, along with constant lines showing its mean and two standard deviations. On the left, the user is brushing observations that lie beyond two standard deviations from the mean. On the right, the user has eliminated these extreme values by selecting **Brushing > Remove brushed**

from the **Tools** (or context) menu. The plot immediately redisplay with three fewer x- and y-values. The original workspace variable, however, remains unchanged.



Before removing the extreme values, you can save them as a new workspace variable with **Tools > Brushing > Create new variable**. Doing this opens a dialog box for you to declare a variable name.



Typing `extremevals` to name the variable and pressing **OK** to dismiss the dialog produces

```
extremevals =
```

```
    9.0000    3.5784
   12.0000    3.0349
   35.0000   -2.9443
```

The new variable contains one row per observation selected. The first column contains the x-values and the second column contains the y-values, copied from the lineseries' XData and YData. In graphs where multiple series are brushed, the Create New Variable dialog box helps you identify what series the new variable should represent, allowing you to select and name one at a time.

### Effects of Brushing on Data

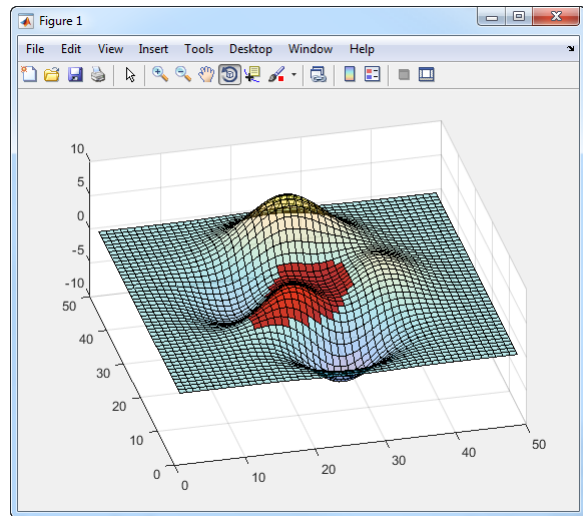
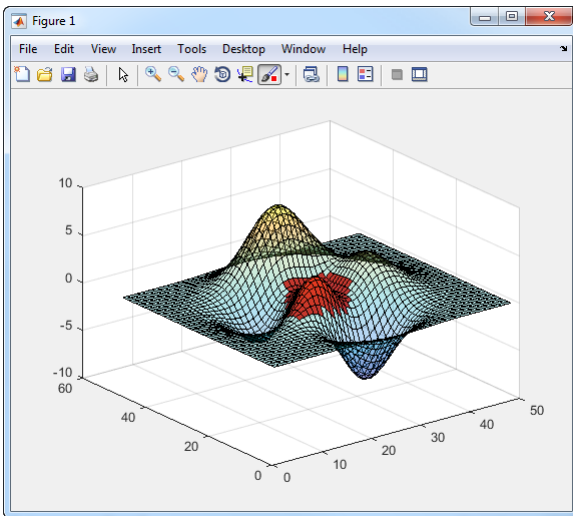
Brushing simply highlights data points in a graph, without affecting data on which the plot is based. If you remove brushed or unbrushed observations or replace them with NaN values, the change applies to the XData, YData, and possibly ZData properties of the plot itself, but not to variables in the workspace. You can undo such changes. However, if you replot a brushed graph using the same workspace variables, not only do its brushing marks go away, all removed or replaced values are restored and you cannot undo it. If you want brushing to affect the underlying workspace data, you must *link* the plot to the variables it displays. See “Making Graphs Responsive with Data Linking” on page 2-12 for more information.

### Brushed 3-D Plots

When an axes displays three-dimensional graphics, brushing defines a region of interest (ROI) as an unbounded rectangular prism. The central axis of the prism is a line perpendicular to the plane of the screen. Opposite corners of the prism pass through points defined by the CurrentPoint associated with the initial mouse click and the value of CurrentPoint during the drag. All vertices lying within the rectangular prism ROI highlight as you brush them, even those that are hidden from view.

The next figure contains two views of a brushed ROI on a peaks surface plot. On the left plot, only the cross-section of the rectangular prism is visible (the brown rectangle) because the central axis of the prism is perpendicular to the viewing plane. When the viewpoint rotates by about 90 degrees clockwise (right-hand plot), you see that the prism extends along the initial axis of view and that the brushed region conforms to the surface.





### Brushed Multiple Plots

When the same  $x$ -,  $y$ - or  $z$ -variable appears in several plots, brushing observations in one plot highlights the related observations in the other plots when they are linked. If the brushed variables are open in the Variables editor, the rows containing the brushed observations are highlighted. For more information, see “Data Brushing with the Variables Editor” on page 2-22.

### Organizing Plots for Brushing

Data brushing usually involves creating multiple views of related variables on graphs and in tables. Just as computer users organize their virtual desktops in many different ways, you can use various strategies for viewing sets of plots:

- Multiple overlapping figure windows
- Tiled figure windows
- Tabbed figure windows
- Subplots presenting multiple views

When MATLAB figures are created, by default, they appear as separate windows. Many users keep them as such, arranging, overlapping, hiding and showing them as their work requires. Any figure, however, can dock inside a figure group, which itself can float or dock in the MATLAB desktop. Once docked in a figure group, you can float and overlap

the individual plots, tile them in various arrangements, or use tabs to show and hide them.

---

**Note** For more information on managing figure windows, see “Document Layout”.

---

Another way of organizing plots is to arrange them as subplots within a single figure window, as illustrated in the example for “Linking vs. Refreshing Plots” on page 2-17. You create and organize subplots with the `subplot` function. Subplots are useful when you have an idea of how many graphs you want to work with simultaneously and how you want to arrange them (they do not need to be all the same size).

---

**Note** You can easily set up MATLAB code files to create subplots; see `subplot` for more information.

---

### Other Data Brushing Aspects

Not all types of graphs can be brushed, and each type that you can brush is marked up in a particular way. To be brushable, a graphic object must have `XDataSource`, `YDataSource`, and where applicable, `ZDataSource` properties. The one exception is the objects produced by the `histogram` and `histogram2` functions, which are brushable due to the special handling they receive. In order to brush a histogram, you must put the figure containing it into a linked state.

The `brush` function reference page explains how to apply brushing to different graph types, describes how to use different mouse gestures for brushing, and lists graph types that you can brush. See the following sections:

- “Types of Charts You Can Brush”
- “Mouse Gestures for Data Brushing”

Keep in mind that data brushing is a mode that operates on entire figures, like zoom, pan, or other modes. This means that some figures can be in data brushing mode at the same time other figures are in other modes. When you dock multiple figures into a figure group, there is only one toolbar, which reflects the state or mode of whatever figure docked in the group you happen to select. Thus, even when docked, some graphs may be in data brushing mode while others are not.

If an axes contains a plot type that cannot be brushed, such as an image object, you can select the figure's Data Brushing tool and trace out a rectangle by dragging it, but no brush marks appear. When you lay out graphs in subplots within a single figure and enter data brushing mode, all the subplot axes become brushable as long as the graphic objects they contain are brushable. If the figure is also in a linked state, brushing one subplot marks any other in the figure that shares a data source with it. Although this also happens when separate figures are linked and brushed, you can prevent individual figures from being brushed by unlinking them from data sources.

## Making Graphs Responsive with Data Linking

### In this section...

“What Is Data Linking?” on page 2-12  
“Why Use Linked Plots?” on page 2-13  
“How to Link Plots” on page 2-13  
“How Linked Plots Behave” on page 2-14  
“Linking vs. Refreshing Plots” on page 2-17  
“Using Linked Plot Controls” on page 2-19

### What Is Data Linking?

Linked plots are graphs in figure windows that visibly respond to changes in the current workspace variables they display and vice versa. This differs from the default behavior of graphs, which contain copies of variables they represent (their `XData/YData/ZData`) and must be explicitly replotted in order to update them when a displayed variable changes. For example, if variable `y` in the workspace appears in a linked plot and `y` is modified in the Command Window, the graphic representation of `y` in the linked plot updates within half a second to reflect the change.

If you use the Variables editor, you might be familiar with data linking. When variables change or go out of scope, the Variables editor updates itself. It continuously updates variables in the workspace when you add, change, or delete values. The Variables editor works the same way with linked plots.

You can programmatically update a plot after the elements in one variable change. For example, the following code calls `refreshdata` to update the plot after `y` changes.

```
x = 0:.1:8*pi;  
y = sin(x);  
h = plot(x,y)  
set(h,'XDataSource','x');  
set(h,'YDataSource','y');  
y = sin(x.^3);  
refreshdata
```

For more information on this manual technique, see the `refreshdata` reference page. Prior to data linking, you need to explicitly update your plots to reflect changes in your workspace variables, as illustrated in “Linking vs. Refreshing Plots” on page 2-17.

## Why Use Linked Plots?


If the same variable appears in plots in multiple figures, you can link any of the plots to the variable. You can use linked plots in concert with “Marking Up Graphs with Data Brushing” on page 2-4, but also on their own. Linking plots lets you

- Make graphs respond to changes in variables in the base workspace or within a function
- Make graphs respond when you change variables in the Variables editor and Command Line
- Modify variables through data brushing that affect different graphical representations of them at once
- Create graphical “watch windows” for debugging purposes

Watch windows are useful if you program in the MATLAB language. For example, when refining a data processing algorithm to step through your code, you can see graphs respond to changes in variables as a function executes statements.

## How to Link Plots


When you create a figure, by default, data linking is off. You can put a figure into a linked state in any of three ways:

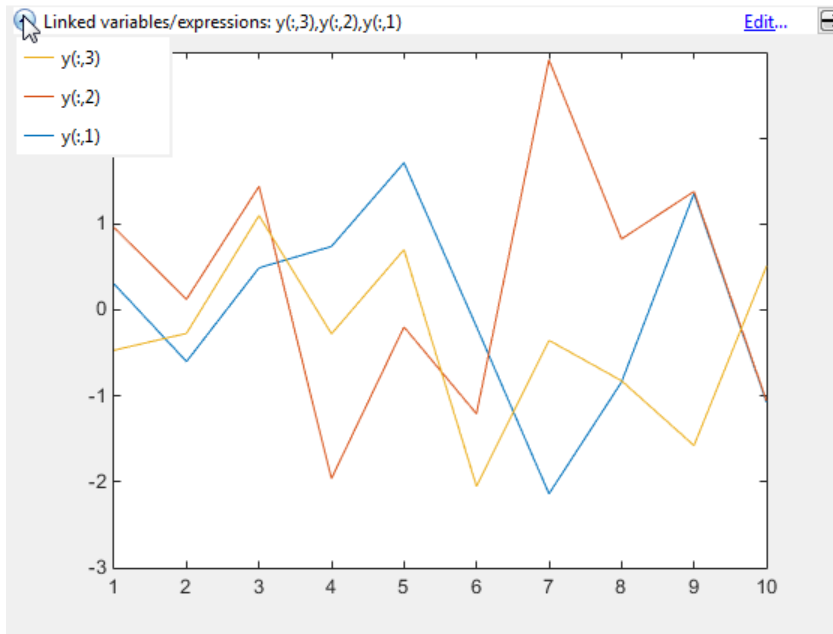
- Click the Data Linking tool button  on the figure toolbar.
- Select **Link** from the figure **Tools** menu.
- Call the `linkdata` MATLAB function, e.g., `linkdata on`.
- To disable data linking, click the Data Linking tool button, deselect **Tools > Link**, or type `linkdata off`.

Once a figure is linked, its appearance changes; an information bar, called the linked plot information bar, appears beneath the figure toolbar to reflect its new linked state. It identifies all linked variables and gives you an opportunity to unlink or relink any of them. The linked plot information bar identifies a figure as being linked and displays relationships between graphic objects and the workspace variables they represent. Click the circular down arrow icon on its left side to display a legend that identifies the data source for each graphic object in a graph.

For example, execute this code at the command line:

```
y = randn(10,3);  
plot(y)
```

Then, click the Data Linking tool button , and click the circular down arrow icon on the left side of the linked plot information bar.



Dropping down the linked plot legend is useful when many data sources are linked to a graph at once. Like legends created with the `legend` function, it identifies graph components with variable expressions.


### How Linked Plots Behave

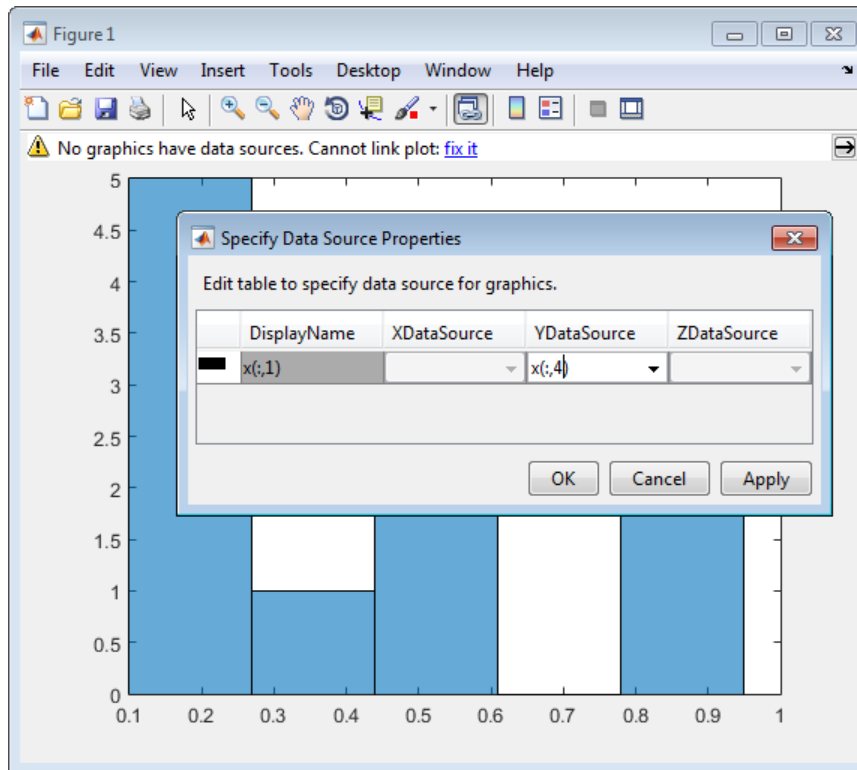
Once linked to its data source(s), a figure acts as if you called the MATLAB function `refreshdata` every time a workspace variable it displays changes. That is, any series or group graphic objects contained in the figure can update its own `XData`, `YData`, or `ZData` properties and redraw itself when one of its data sources is modified. If the linked state is set to 'off' using the `linkdata` function, by deselecting the **Data Linking** toolbar button, or by deselecting **Link** on the figure's **Tools** menu, automatic refreshing stops.

When you turn linking on for a figure, the linking mechanism can usually identify the data sources for displayed graphs, but sometimes ambiguity exists about what variable or range of a variable has been plotted. At such times, the Linked Plot information bar informs you that graphics have no data sources and gives you a chance to identify them. Click **fix it** to open a dialog box where you can specify the variables and ranges of any or all plotted variables.

For example, create a matrix of random data and plot a histogram of the fourth column using five bins.

```
x = rand(10);  
histogram(x(:,4),5)
```

Click the Data Linking tool button  and then click **fix it** in the Linked Plot information bar. Use the drop down menu under YDataSource to select the option `x(:, 1)`. Edit the column index from 1 to 4 and select **OK**.




---

**Note** You can create graphs that have no data sources. For example, `plot(randn(100,1))` generates a line graph that has neither an `XDataSource` (the `x`-values are implicit) nor a `YDataSource` (no variable for `y`-values exists). Therefore, while you can brush such graphs, you cannot link them to data sources, because linking requires workspace data. Similarly, if you create a variable, graph it, and then clear the variable from the workspace you will be unable to link that plot.

---

When you brush a graph that is not linked to data sources, you brush the graphics only. The brushing affects only the figure you interact with. However, when you brush a linked plot, you are brushing the underlying variables. In this case, your brush marks also display on all linked plots that have the same data sources you brushed, as well as any display of that data which you have opened in the Variables editor. The color of the brush marks in all displays is the brush color you have selected for the figure in which you are



brushing. This color can differ from the brush colors you have chosen to use in others display, and overrides those colors.

## Linking vs. Refreshing Plots

Besides the linked plots feature, other MATLAB mechanisms connect graphic objects to data sources (workspace variables). The main techniques are:

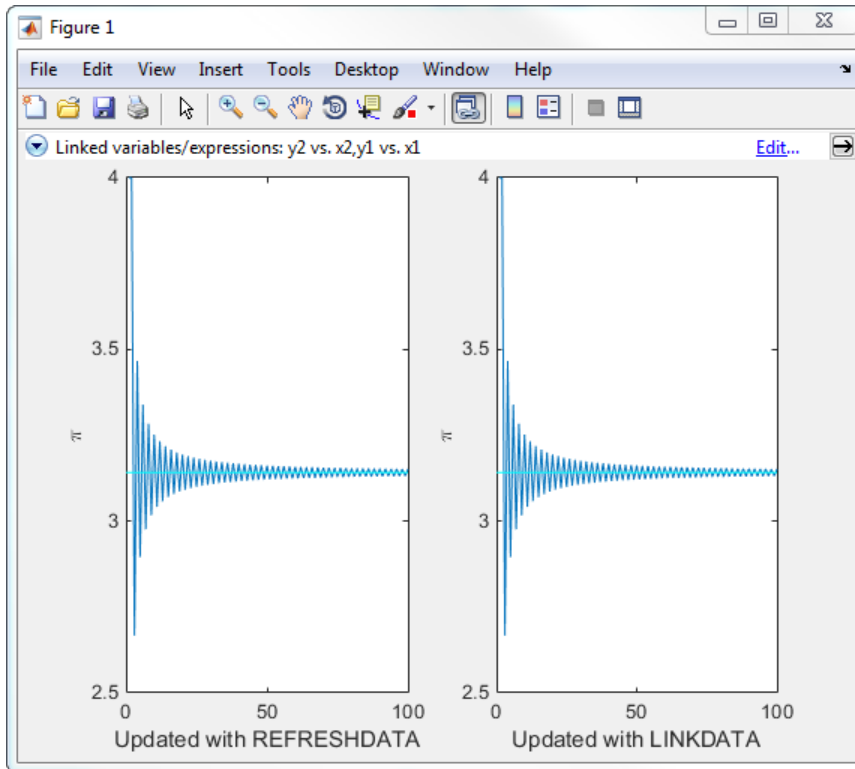
- Directly update the `XData/YData/ZData` properties of a graph.
- Set a graph's `XDataSource/YDataSource/ZDataSource` and indirectly update `XData/YData/ZData` by calling `refreshdata`.

For an example of updating object properties to animate graphics, see “Trace Marker Along Line”. Data linking is not a method intended for animating data graphs.


Linking plots automates these tasks and keeps graphs continuously in sync with the variables they depict, making it the easiest technique to use. Data sources must still exist in the workspace, but you do not need to explicitly declare them for linked plots unless some ambiguity exists. The following code examples iteratively approximate  $\pi$ , and illustrate the difference between declaring and refreshing data sources yourself and letting the `linkdata` function handle it for you.

Updating a Graph with refreshdata	Updating a Graph with linkdata
<pre> x1= [1 2]; y1 = [4 4]; ntimes = 100; denom = 1; k = -1; subplot(1,2,1) hp1 = plot(x1,y1); xlabel('Updated with REFRESHDATA') ylabel('\pi') set(gca,'Xlim',[0 ntimes],...       'Ylim',[2.5 4]) set(hp1,'XDataSource', 'x1') set(hp1,'YDataSource', 'y1') for t = 3:ntimes     denom = denom + 2;     x1(t) = t;     y1(t) = 4*(y1(t-1)/4 + k/denom);     refreshdata     drawnow     k = -k; end line([0 ntimes], [pi pi],'color','c') </pre>	<pre> x2= [1 2]; y2 = [4 4]; ntimes = 100; denom = 1; k = -1; subplot(1,2,2) plot(x2,y2); xlabel('Updated with LINKDATA') ylabel('\pi') set(gca,'Xlim',[0 ntimes],...       'Ylim',[2.5 4]) linkdata on for t = 3:ntimes     denom = denom + 2;     x2(t) = t;     y2(t) = 4*(y2(t-1)/4 + k/denom);     k = -k; end line([0 ntimes], [pi pi],'color','c') </pre>


Differences are shown in italics. When you execute the code on the left, which uses `refreshdata`, it animates the approximation process. The code on the right uses `linkdata` and does not animate; it runs much faster. (A `drawnow` command is not needed, because data linking buffers update and refresh the graph at half-second intervals.) The graphic results, shown in the next image, are identical. Because both plots are in axes in the same figure, linking the second graph also links the first graph to its variables.

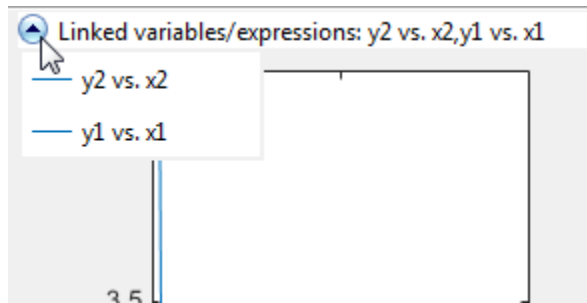


## Using Linked Plot Controls

To minimize the Linked Plot information bar while remaining in linked mode, click the hide/show button  on its right side; the button flips direction and the bar is hidden. Clicking the button again flips the arrow back and restores the Linked Plot information bar. Turning off linking cuts all data source connections and removes the Linked Plot information bar from the figure. However, the data source properties remain set, and the bar reappears whenever a linked state is restored by selecting **Tools > Link**, depressing the **Linked Plot** button, or calling the `linkdata` function. Whatever data sources were established previously will then reconnect (assuming those variables still exist in the same form).

## The Data Source Button

The  down arrow button on the left side of the Linked Plot information bar drops down a legend (similar to what the `legend` function produces but without Display Names). The legend identifies workspace variables associated with plot objects for the entire figure (Legend works on a per-axis basis), such as these linked lineseries from the previous example, shown in the next image.



The drop-down legend names variable linked to the graphic objects in the figure. For items to appear there, a graph must have an `XDataSource`, `YDataSource`, or a `ZDataSource` property that MATLAB can evaluate without error. The icon for each list entry reflects the `Color`, `LineStyle` and `Marker` of the corresponding graphic object, making clear which graphic objects link to which variables. The drop-down legend is informational only; you can only dismiss it after reading it by clicking anywhere else on the figure.

## The Edit Button



Clicking the **Edit** link on the information bar opens the Specify Data Source Properties modal dialog box for you to set the `DisplayName`, `XDataSource`, `YDataSource`, and `ZDataSource` properties of plot objects in the figure to columns or vectors of workspace variables. Changing a `DisplayName` updates text on a legend, if present for the variable, and has no other effects. The three columns on the right contain drop-down lists of workspace variables. You can also type variable names and ranges, or a MATLAB expression. When you change variables or their ranges on the fly with this dialog box, variables plotted against one another must be compatible types and have the same number of observations (as in any bivariate graph).

If you attempt to link a plot and `linkdata` can identify more than one possible workspace variable for one or more plot objects, the Specify Data Source Properties dialog box

appears for you to resolve the ambiguity. If you choose not to or are unable to do so and cancel the dialog box, data linking is not established for those graphic objects.

### **When Data Links Fail**

Updating a linked plot can fail if the `XDataSource`, `YDataSource`, or `ZDataSource` property values are incompatible with what is in the current workspace. Consequently, the corresponding `XData`, `YData`, and `ZData` cannot be updated. This happens most often because variables are cleared or no longer exist when the workspace changes (e.g., when you are debugging).

However, failing links do not affect the visual appearance of the object in the graph. Instead, a warning icon and message appears on the Linked Plot information bar when this occurs for any plotted data in the figure. The failing link warning is general, but you can identify which variables are affected by clicking the Data Source  button. If you hide the Linked Plot information bar (by clicking its Hide  button), the bar reappears when a data links fails, alerting you to the issue.

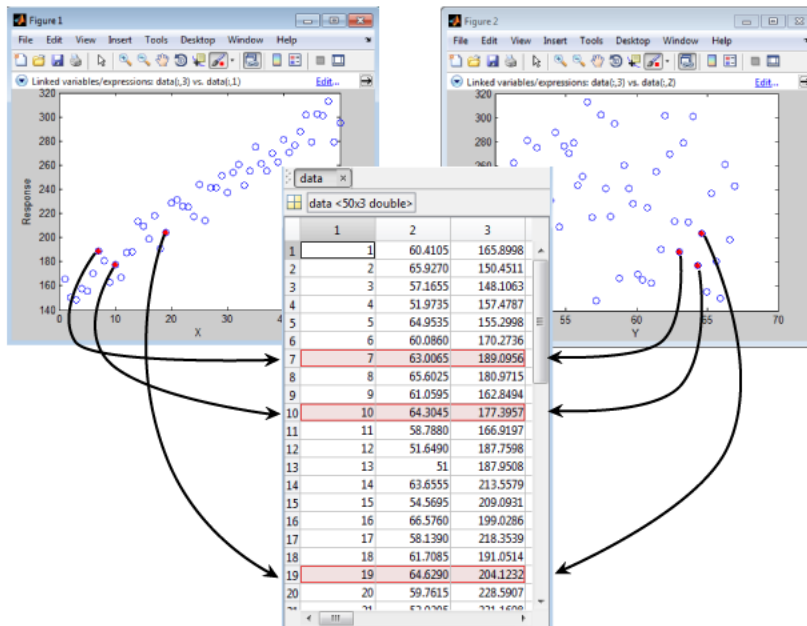
## Interacting with Graphed Data

<b>In this section...</b>
“Data Brushing with the Variables Editor” on page 2-22
“Using Data Tips to Explore Graphs” on page 2-23
“Example — Visually Exploring Demographic Statistics” on page 2-24

### Data Brushing with the Variables Editor

To brush data in the Variables editor, link the figure windows associated with variable. Then right-click on a cell in the Variables editor and select **Brushing > Brushing on** in the context menu. Select one or more cells to brush elements in the variable. The corresponding points on your plots highlight simultaneously.


You can brush observations that appear in multiple linked plots at the same time. You can do this only when your observations are in a matrix with the plot variables running along separate columns. For example, you can create two separate plots of observations in a matrix called `data`, which contains system response measurements at 50 different  $(x, y)$  points. The first column, `data(:, 1)`, contains the  $x$ -coordinates, `data(:, 2)` contains  $y$ -coordinates, and `data(:, 3)` contains the measured response at each point. The left plot below shows the response versus  $x$ . The plot on the right shows the response versus  $y$ . If you brush a point in one plot, the corresponding point in the other plot highlights at the same time. Furthermore, if you have the Variables editor open, the corresponding data row is highlighted whenever you brush a point.



For more information about the using the Variables editor, see the [openvar](#) reference page.

## Using Data Tips to Explore Graphs

A data tip is a small display associated with an axes that reads out individual data observation values from a 2-D or 3-D graph. You create data tips by mouse clicks on

graphs using the **Data Cursor tool**  from the figure toolbar. When you select this tool, you are in data cursor mode—signified by a hollow cross-hair cursor—in which you identify  $x$ -,  $y$ -, and  $z$ -values of data points you click. Like data points you brush, export such values to the workspace.

For descriptions of data cursor properties and how to use them, see

- “Display Data Values Interactively” and “Data Cursors with Histograms”
- The MATLAB function reference page for `datacursormode`

The default behavior of data tips is to simply display the `XData`, `YData`, and `ZData` values of the selected observations as text in a box. Sometimes this information is not helpful by itself, and you might want to replace or augment it with other information. You can modify this behavior to display other facts connected to observations. You customize data tip behavior by constructing a data tip text update function (in MATLAB code) to construct text for display in data tips and then instructing data cursor mode to use your function instead of the default one.

Customize data cursor update functions to display information such as

- Names associated with  $x$ -,  $y$ -, and  $z$ -values
- Weights associated with  $x$ -,  $y$ -, and  $z$ -values
- Differences in  $x$ -,  $y$ -, and  $z$ -values from the mean or their neighbors
- Transformations of values (e.g., normalizations or to different units of measure)
- Related variables

You can create data tip text update functions to display such information and change their behavior on the fly. You can even make the update function behave differently for distinct observations in the same graph if your update function or the code calling it can distinguish groups of them. The next section contains an example of coding and using a customized data cursor update function.

### **Example — Visually Exploring Demographic Statistics**

- “The Data Tip Text Update Function” on page 2-25
- “Preparing, Plotting, and Annotating the Data” on page 2-26
- “Explore the Graph with the Custom Data Cursor” on page 2-29
- “Plot and Link a Histogram of a Related Variable” on page 2-31
- “Explore the Linked Graphs with Data Brushing” on page 2-32
- “Plot the Observations on a Linked Map” on page 2-33

The extended example that follows begins by using data tips to explore the incidence of fatal traffic accidents tabulated for U.S. states, with respect to state populations. The example extends this analysis to brush, link, and map the data to discover spatial patterns in the data. Each section of the example has four or fewer steps. By executing them all, you gain insight into the data set and become familiar with useful graphical data exploration techniques.



Censuses of population and other national government statistics are valuable sources of demographic and socioeconomic data. An important aspect of census data is its geography, i.e., the regions to which a given set of statistics applies, and at what level of granularity. When exploring census data, you frequently need to identify what geographic unit any given observation represents.

This example uses data tips to show place names and statistics for individual observations. You pass place names and the data matrix to a custom text update function to enable this. The place names are for U.S. states and the District of Columbia. If all these names were placed as labels on the x-axis, they would be too small or too crowded to be legible, but they are readable one at a time as data tips.

The example also illustrates how sorting a data matrix by rows can enhance interpretation when the original ordering (in this case alphabetical by state) provides no special insight into relationships among observations and variables.

### The Data Tip Text Update Function

Data tips can present other information beyond x-, y- and z-values. Read through the example function `labeldtips`, which takes three more parameters than a default callback, and displays the following information:

- Its y-value
- Deviation from an expected y-value
- Percent deviation from the expected y-value
- The observation's label (state name)

Because it customizes data tips, the function must be a code file that you invoke from the Command Window or from a script. This file, `labeldtips.m`, and the MAT-files `accidents.mat` and `usapolygon.mat` that the following examples also use, exist on the MATLAB path. Here is the code for the `labeldtips` data cursor callback function.

```
function output_txt = labeldtips(obj,event_obj,...
                               xydata,labels,xymean)
% Display an observation's Y-data and label for a data tip
% obj           Currently not used (empty)
% event_obj    Handle to event object
% xydata       Entire data matrix
% labels       State names identifying matrix row
% xymean       Ratio of y to x mean (avg. for all obs.)
% output_txt   Datatip text (character vector or cell array)
```

```
%           of character vectors)
% This datacursor callback calculates a deviation from the
% expected value and displays it, Y, and a label taken
% from the cell array 'labels'; the data matrix is needed
% to determine the index of the x-value for looking up the
% label for that row. X values could be output, but are not.

pos = get(event_obj, 'Position');
x = pos(1); y = pos(2);
output_txt = [{'Y: ', num2str(y,4)}];
ydev = round((y - x*xymean));
ypct = round((100 * ydev) / (x*xymean));
output_txt{end+1} = ['Yobs-Yexp: ' num2str(ydev) ...
                   '; Pct. dev: ' num2str(ypct)];
idx = find(xydata == x,1); % Find index to retrieve obs. name
% The find is reliable only if there are no duplicate x values
[row,col] = ind2sub(size(xydata),idx);
output_txt{end+1} = cell2mat(labels(row));
```

The portion of the example called “Explore the Graph with the Custom Data Cursor” on page 2-29 sets up data cursor mode and declares this function as a callback using the following code:

```
hdt = datacursormode;
set(hdt, 'UpdateFcn', {@labeldtips, hwydata, statelabel, usmean})
```

The call to `datacursormode` puts the current figure in data cursor mode. `hdt` is the handle of a data cursor mode object for the figure you want to explore. The function name and its three formal arguments are a cell array.

### Preparing, Plotting, and Annotating the Data

The following steps show how you load statistical data for U.S. states, plot some of it, and enter data cursor mode to explore the data:

---

**Note** To help you interpret graphs created in this example, the `hwydata` data matrix and its row labels have been presorted by rows to be in ascending order by total state population. The 51-by-1 vector `hwyidx` contains indices from the presorting (the data were originally in alphabetic order)

If you ever want to resort the data array and state labels alphabetically, you can sort on the first column of the `hwydata` matrix, which contains Census Bureau state IDs that ascend in alphabetical order, as follows:

```
[hwydata hwyidx] = sortrows(hwydata,1);
statelabel = statelabel(hwyidx);
```

If you do resort the data, to make the graph easier to interpret you might plot it using markers rather than lines. To do this, change the call to `plot` in section 2, below, to the following:

```
plot(hwydata(:,14),hwydata(:,4),'.')
```

- 1 Load U.S. state data statistics from the National Transportation Safety Highway Administration and the Bureau of the Census and look at the variables:

```
load 'accidents.mat'
```

```
whos
```

Name	Size	Bytes	Class
datasources	3x1	2568	cell
hwycols	1x1	8	double
hwydata	51x17	6936	double
hwyheaders	1x17	1874	cell
hwyidx	51x1	408	double
hwyrows	1x1	8	double
statelabel	51x1	3944	cell
ushwydata	1x17	136	double
uslabel	1x1	86	cell

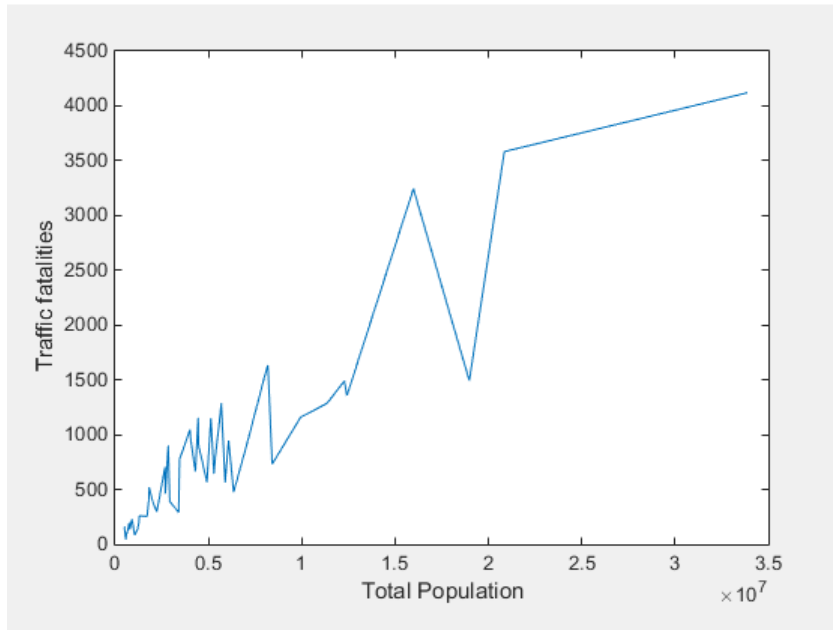
The data set has 51 observations for 17 variables.

- The state-by-state statistics; the double 51-by-17 matrix `hwydata`
  - The variable (column) names; the 1-by-17 text cell array `hwyheaders`
  - The state names; the 51-by-1 text cell array `statelabel`
  - Values for the entire United States for the 17 variables; the 1-by-17 matrix `ushwydata`
  - The label for the US values; the 1-by-1 cell array `uslabel`
  - Metadata describing data sources; the 3-by-1 cell array `datasources`
- 2 Plot a line graph of the population by state as  $x$  versus the number of traffic fatalities per state as  $y$ :

```
hf1 = figure;
plot(hwydata(:,14),hwydata(:,4));
```

```
xlabel(hwyheaders(14))
ylabel(hwyheaders(4))
```

Because the state observations are sorted by population size, the graph is monotonic in  $x$ . The larger a population a state has, the more variation in traffic accident fatalities it tends to show.



- 3 Compute the per capita rate of traffic fatalities for the entire United States; in the next part of this example, the data cursor update function uses this average to compute an expected value for each state you query:

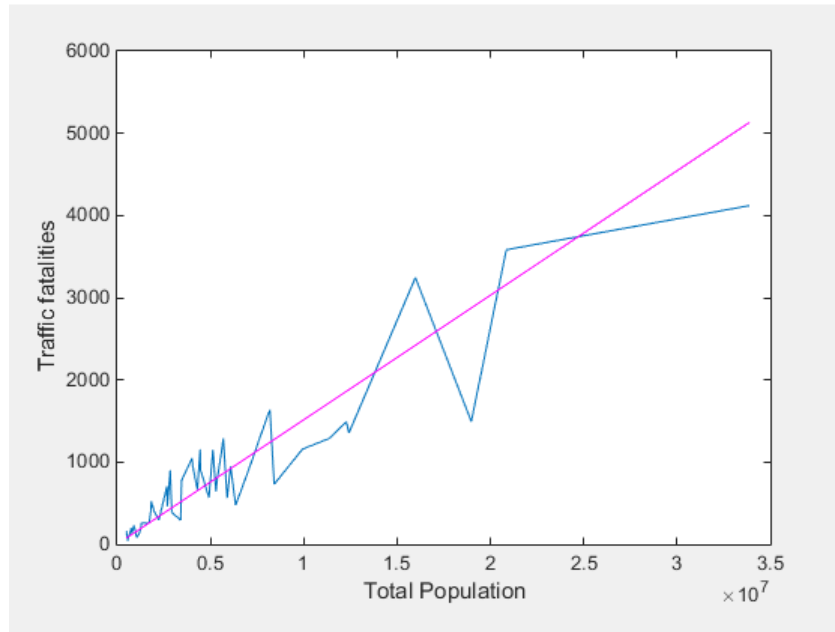
```
usmean = ushwydata(4)/ushwydata(14)
```

```
usmean =
  1.5150e-004
```

The statistic shows that nationally, about 150 per 100,000 people die in traffic accidents every year.

Use `usmean` to compute the smallest and largest expected values by multiplying it by the smallest and largest state populations, and draw a line connecting them:

```
line([min(hwydata(:,14)) max(hwydata(:,14))],...
     [min(hwydata(:,14))*usmean max(hwydata(:,14))*usmean]),...
     'Color','m');
```




---

**Note** The magenta line is not a regression line; it is a trend line that plots the number of traffic deaths that a state of a given size would have if all states obeyed the national average.

---

### Explore the Graph with the Custom Data Cursor

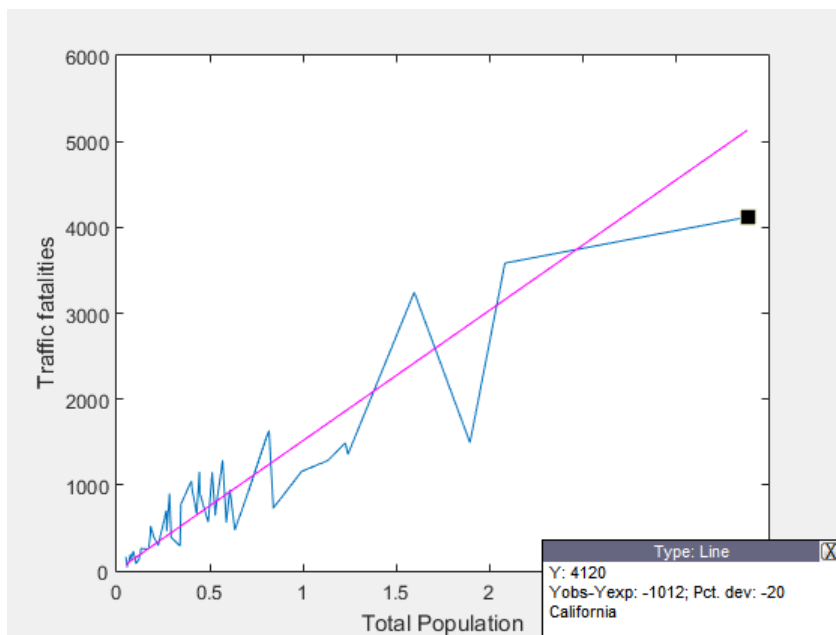
You can now explore the graphed data with the example custom data cursor update function on page 2-25 `labeldtips` (which must be on the MATLAB path or in the current folder). `labeldtips` displays state names and y-deviations.

- 1 Turn on data cursor mode and invoke the custom callback:

```
hdt = datacursormode;
set(hdt,'DisplayStyle','window');
% Declare a custom datatip update function
```

```
% to display state names:
set(hdt, 'UpdateFcn', {@labeldtips, hwydata, statelabel, usmean})
```

The data cursor 'window' display style sends data tip output to a small window that you can move anywhere within the figure. This display style is best suited to data tips that contain more text than just  $x$ -,  $y$ -, and  $z$ -values. The `labeldtips` callback remains active for that figure until you use `set` to replace it with another function (or empty, to restore the default data cursor behavior). Click the right-most point on the blue graph.



The data tip shows that California has the largest population and the largest number of traffic fatalities, 4120. However, it had 1012, or 20%, fewer fatalities than predicted by the national average. The next data point to the left depicts Texas. Click that data point or press the left arrow to show its data tip. To see results from other states, move the data tip by dragging the black square or using the left or right arrow to step it along the graph. If you know a little about U.S. geography, you might observe a pattern.

## Plot and Link a Histogram of a Related Variable


The ninth column of `hwydata`, labeled "Fatalities per 100K Licensed Drivers," is related to population. Plot a histogram of this variable to see which states have fewer or more fatalities per driver. To do this, link the plots to their data, and brush either of them.

- 1 Open a new figure and plot a histogram of Fatalities per 100K Licensed Drivers using five bins:

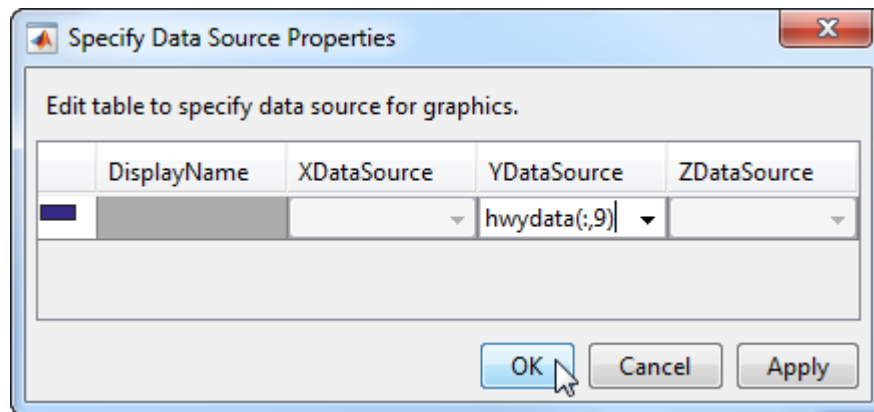
```
hf2 = figure
histogram(hwydata(:,9),5)
xlabel(hwyheaders(9))
```

- 2 Link both the line graph and the histogram to their data sources in `hwydata`:

```
linkdata(hf1)
linkdata(hf2)
```

You can also click the **Link Plot** tool  on either figure. The first figure links automatically, however the histogram does not because `linkdata` cannot determine with certainty the `YDataSource` for histograms. The Linked Plot information bar on top of the histogram informs you **No Graphics have data sources. Cannot link plot: fix it.**


- 3 Click **fix it** to open the Specify Data Source Properties dialog box. Type `hwydata(:,9)` into the `YDataSource` edit box and click **OK**.

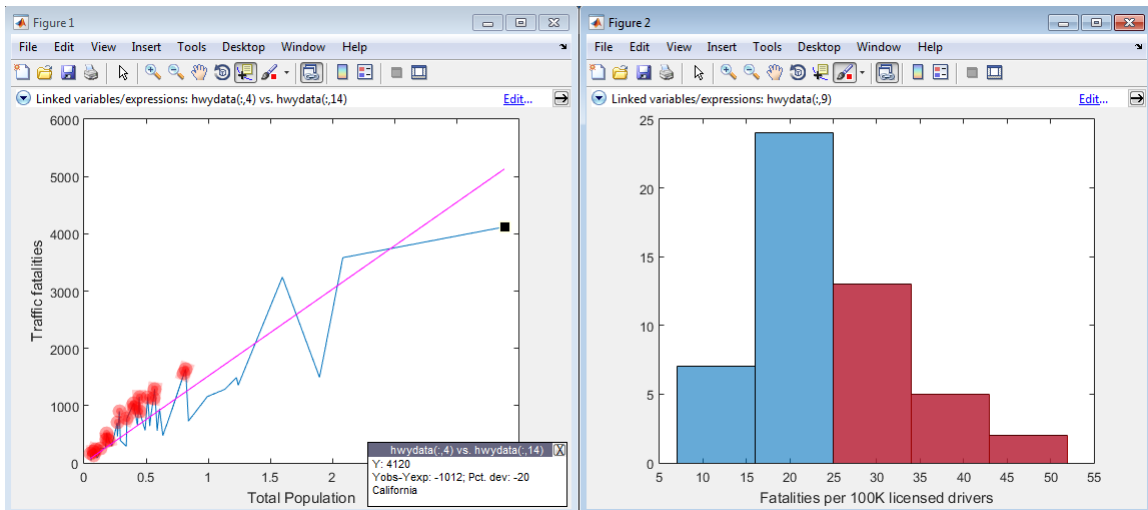


The Linked Plot information bar displays the data source you identified.

### Explore the Linked Graphs with Data Brushing

Now that you have linked both graphs to a common data set, you can brush portions of one to see the effect on the other.

- 1 Arrange both figures on your computer screen so that they are simultaneously visible.
- 2 Select the Data Brushing tool  on the histogram plot. Brush the three right-most bars in the histogram by clicking the third bar and dragging to the right over the last two bars. The brushed bars represent the number of fatalities ranging from 25 to 52 per 100,000 drivers.



Notice the data points that are automatically highlighted on the line graph. These points generally correspond to the states with smaller populations, but above-average traffic fatality totals.

- 3 Click the line graph to make it the active figure and select its Data Brushing tool. Select all of the points falling *below* the straight-line average by holding the **Shift** key and either clicking or dragging your selections. It might be helpful to zoom in on the graph to do this. What do you see happening on the histogram?



## Plot the Observations on a Linked Map

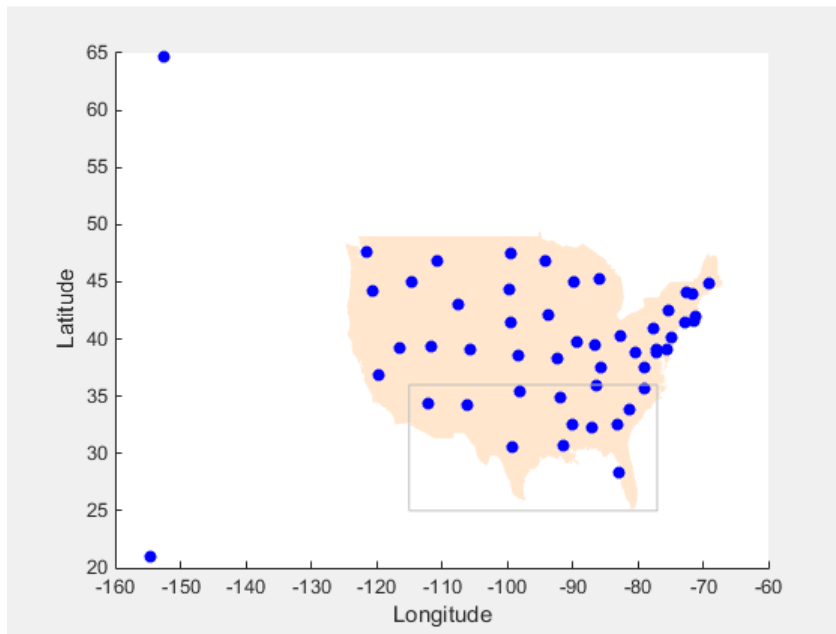
The `hwydata` matrix contains geographic location information in the form of latitude-longitude coordinates of a centroid for each state. You can make a crude map by generating a scatter plot of these coordinates, using longitude as  $x$  and latitude as  $y$ . If you link the scatter plot, you can brush all the plots at once.

- 1 To provide a context for the map, plot an outline map of the conterminous United States. Obtain the latitude and longitude coordinates required from the MAT-file `usapolygon.mat`:

```
hf3 = figure;  
load usapolygon  
patch(uslon,uslat,[1 .9 .8],'Edgecolor','none');  
hold on
```

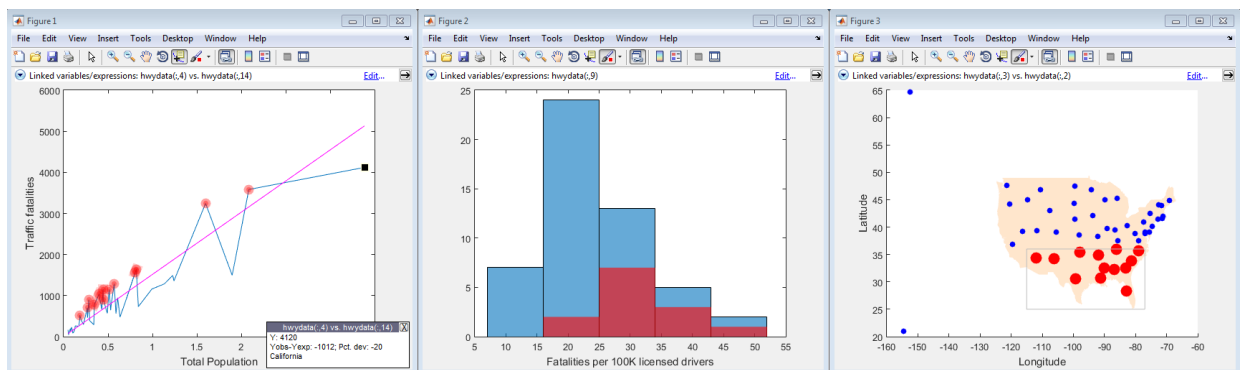
- 2 Map the centroid longitude and latitude as a scatter plot with filled circles. Plot a rectangle over part of the map, as follows:

```
scatter(hwydata(:,2),hwydata(:,3),36,'b','filled');  
xlabel('Longitude')  
ylabel('Latitude')  
rectangle('Position',[-115,25,115-77,36-25],...  
         'EdgeColor',[.75 .75 .75])
```



The x- and y-limits change, shrinking the map, because the data matrix contains observations for Alaska and Hawaii, but the map outline file does not include these states.

- Align the map alongside the other two figures. Brush the map after turning on the Data Linking and Data Brushing tools. Drag across the gray rectangle with the Data Brushing tool to highlight the southern-most states. The result should look like this:



Data brushing and linking reveals that most of the states with above-average traffic fatality totals are in the southern part of the U.S.

Using graphic data exploration, you have identified some intriguing regularities in this data. However, you have not identified any causes for the patterns you found. That will take more work with the data, and possibly additional data sets, along with some hypotheses and models.



# Regression Analysis

---

- “Linear Correlation” on page 3-2
- “Linear Regression” on page 3-6
- “Interactive Fitting” on page 3-16
- “Programmatic Fitting” on page 3-35

## Linear Correlation

### In this section...

“Introduction” on page 3-2

“Covariance” on page 3-3

“Correlation Coefficients” on page 3-4

### Introduction

*Correlation* quantifies the strength of a linear relationship between two variables. When there is no correlation between two variables, then there is no tendency for the values of the variables to increase or decrease in tandem. Two variables that are uncorrelated are not necessarily independent, however, because they might have a nonlinear relationship.

You can use linear correlation to investigate whether a linear relationship exists between variables without having to assume or fit a specific model to your data. Two variables that have a small or no linear correlation might have a strong nonlinear relationship. However, calculating linear correlation before fitting a model is a useful way to identify variables that have a simple relationship. Another way to explore how variables are related is to make scatter plots of your data.

*Covariance* quantifies the strength of a linear relationship between two variables in units relative to their variances. Correlations are standardized covariances, giving a dimensionless quantity that measures the degree of a linear relationship, separate from the scale of either variable.

The following three MATLAB functions compute sample correlation coefficients and covariance. These sample coefficients are estimates of the true covariance and correlation coefficients of the population from which the data sample is drawn.

Function	Description
corrcoef	Correlation coefficient matrix
cov	Covariance matrix
xcorr (a Signal Processing Toolbox™ function)	Cross-correlation sequence of a random process (includes autocorrelation)

## Covariance

Use the MATLAB `cov` function to calculate the sample covariance matrix for a data matrix (where each column represents a separate quantity).

The sample covariance matrix has the following properties:

- `cov(X)` is symmetric.
- `diag(cov(X))` is a vector of variances for each data column. The variances represent a measure of the spread or dispersion of data in the corresponding column. (The `var` function calculates variance.)
- `sqrt(diag(cov(X)))` is a vector of standard deviations. (The `std` function calculates standard deviation.)
- The off-diagonal elements of the covariance matrix represent the covariances between the individual data columns.

Here,  $X$  can be a vector or a matrix. For an  $m$ -by- $n$  matrix, the covariance matrix is  $n$ -by- $n$ .

For an example of calculating the covariance, load the sample data in `count.dat` that contains a 24-by-3 matrix:

```
load count.dat
```

Calculate the covariance matrix for this data:

```
cov(count)
```

MATLAB responds with the following result:

```
ans =
  1.0e+003 *
    0.6437    0.9802    1.6567
    0.9802    1.7144    2.6908
    1.6567    2.6908    4.6278
```

The covariance matrix for this data has the following form:

$$\begin{bmatrix} s_{11}^2 & s_{12}^2 & s_{13}^2 \\ s_{21}^2 & s_{22}^2 & s_{23}^2 \\ s_{31}^2 & s_{32}^2 & s_{33}^2 \end{bmatrix}$$

$$s_{ij}^2 = s_{ji}^2$$

Here,  $s^2_{ij}$  is the sample covariance between column  $i$  and column  $j$  of the data. Because the count matrix contains three columns, the covariance matrix is 3-by-3.

---

**Note** In the special case when a vector is the argument of `cov`, the function returns the variance.

---

## Correlation Coefficients

The MATLAB function `corrcoef` produces a matrix of sample correlation coefficients for a data matrix (where each column represents a separate quantity). The correlation coefficients range from -1 to 1, where

- Values close to 1 indicate that there is a positive linear relationship between the data columns.
- Values close to -1 indicate that one column of data has a negative linear relationship to another column of data (*anticorrelation*).
- Values close to or equal to 0 suggest there is no linear relationship between the data columns.

For an  $m$ -by- $n$  matrix, the correlation-coefficient matrix is  $n$ -by- $n$ . The arrangement of the elements in the correlation coefficient matrix corresponds to the location of the elements in the covariance matrix, as described in “Covariance” on page 3-3.

For an example of calculating correlation coefficients, load the sample data in `count.dat` that contains a 24-by-3 matrix:

```
load count.dat
```

Type the following syntax to calculate the correlation coefficients:

```
corrcoef(count)
```

This results in the following 3-by-3 matrix of correlation coefficients:

```
ans =  
    1.0000    0.9331    0.9599  
    0.9331    1.0000    0.9553  
    0.9599    0.9553    1.0000
```



Because all correlation coefficients are close to 1, there is a strong positive correlation between each pair of data columns in the count matrix.

## Linear Regression

In this section...
“Introduction” on page 3-6
“Simple Linear Regression” on page 3-7
“Residuals and Goodness of Fit” on page 3-11
“Fitting Data with Curve Fitting Toolbox Functions” on page 3-15

### Introduction

A data *model* explicitly describes a relationship between predictor and response variables. Linear regression fits a data model that is linear in the model coefficients. The most common type of linear regression is a least-squares fit, which can fit both lines and polynomials, among other linear models.

Before you model the relationship between pairs of quantities, it is a good idea to perform correlation analysis to establish if a linear relationship exists between these quantities. Be aware that variables can have nonlinear relationships, which correlation analysis cannot detect. For more information, see “Linear Correlation” on page 3-2.

The MATLAB Basic Fitting UI helps you to fit your data, so you can calculate model coefficients and plot the model on top of the data. For an example, see “Example: Using Basic Fitting UI” on page 3-18. You also can use the MATLAB `polyfit` and `polyval` functions to fit your data to a model that is linear in the coefficients. For an example, see “Programmatic Fitting” on page 3-44.

If you need to fit data with a nonlinear model, transform the variables to make the relationship linear. Alternatively, try to fit a nonlinear function directly using either the Statistics and Machine Learning Toolbox `nlinfit` function, the Optimization Toolbox™ `lsqcurvefit` function, or by applying functions in the Curve Fitting Toolbox™.

This topic explains how to:

- Perform simple linear regression using the `\` operator.
- Use correlation analysis to determine whether two quantities are related to justify fitting the data.
- Fit a linear model to the data.

- Evaluate the goodness of fit by plotting residuals and looking for patterns.
- Calculate measures of goodness of fit  $R^2$  and adjusted  $R^2$

## Simple Linear Regression

This example shows how to perform simple linear regression using the accidents dataset. The example also shows you how to calculate the coefficient of determination  $R^2$  to evaluate the regressions. The accidents dataset contains data for fatal traffic accidents in U.S. states.

Linear regression models the relation between a dependent, or response, variable  $y$  and one or more independent, or predictor, variables  $x_1, \dots, x_n$ . Simple linear regression considers only one independent variable using the relation

$$y = \beta_0 + \beta_1 x + \epsilon,$$

where  $\beta_0$  is the y-intercept,  $\beta_1$  is the slope (or regression coefficient), and  $\epsilon$  is the error term.

Start with a set of  $n$  observed values of  $x$  and  $y$  given by  $(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ . Using the simple linear regression relation, these values form a system of linear equations. Represent these equations in matrix form as

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}.$$

Let

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}, B = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}.$$

The relation is now  $Y = XB$ .

In MATLAB, you can find  $B$  using the `mldivide` operator as  $B = X \backslash Y$ .

From the dataset `accidents`, load accident data in `y` and state population data in `x`. Find the linear regression relation  $y = \beta_1 x$  between the accidents in a state and the population of a state using the `\` operator. The `\` operator performs a least-squares regression.

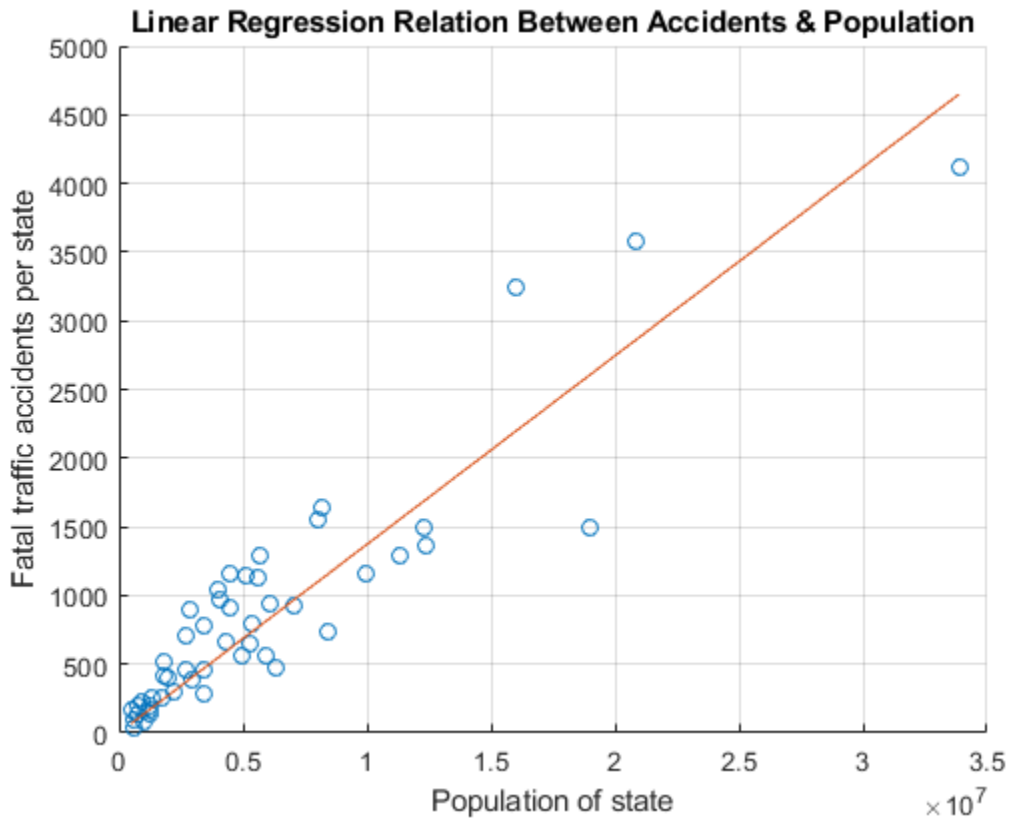
```
load accidents
x = hwydata(:,14); %Population of states
y = hwydata(:,4); %Accidents per state
format long
b1 = x \ y
```

```
b1 =
    1.372716735564871e-04
```

`b1` is the slope or regression coefficient. The linear relation is  $y = \beta_1 x = 0.0001372x$ .

Calculate the accidents per state `yCalc` from `x` using the relation. Visualize the regression by plotting the actual values `y` and the calculated values `yCalc`.

```
yCalc1 = b1*x;
scatter(x,y)
hold on
plot(x,yCalc1)
xlabel('Population of state')
ylabel('Fatal traffic accidents per state')
title('Linear Regression Relation Between Accidents & Population')
grid on
```



Improve the fit by including a y-intercept  $\beta_0$  in your model as  $y = \beta_0 + \beta_1 x$ . Calculate  $\beta_0$  by padding  $x$  with a column of ones and using the  $\backslash$  operator.

```
X = [ones(length(x),1) x];
b = X\y
```

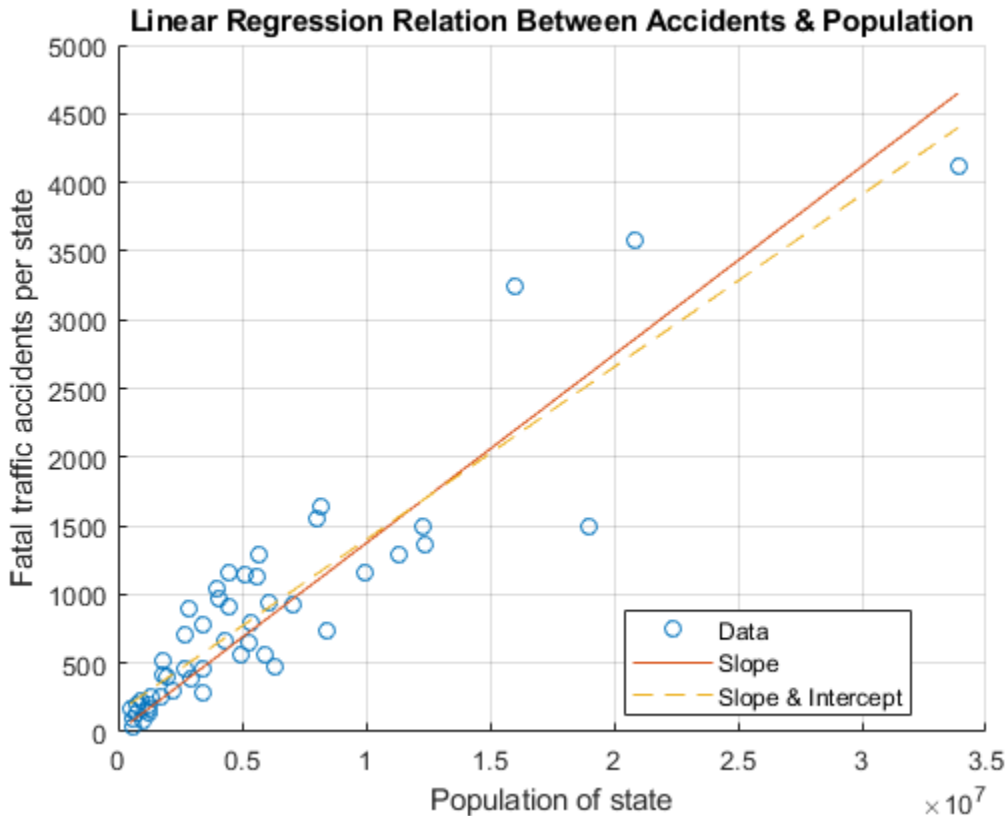
```
b = 2×1
102 ×
```

```
1.427120171726537
0.000001256394274
```

This result represents the relation  $y = \beta_0 + \beta_1x = 142.7120 + 0.0001256x$ .

Visualize the relation by plotting it on the same figure.

```
yCalc2 = X*b;
plot(x,yCalc2,'--')
legend('Data', 'Slope', 'Slope & Intercept', 'Location', 'best');
```



From the figure, the two fits look similar. One method to find the better fit is to calculate the coefficient of determination,  $R^2$ .  $R^2$  is one measure of how well a model can predict the data, and falls between 0 and 1. The higher the value of  $R^2$ , the better the model is at predicting the data.

Where  $\hat{y}$  represents the calculated values of  $y$  and  $\bar{y}$  is the mean of  $y$ ,  $R^2$  is defined as

$$R^2 = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}.$$

Find the better fit of the two fits by comparing values of  $R^2$ . As the  $R^2$  values show, the second fit that includes a y-intercept is better.

```
Rsqr1 = 1 - sum((y - yCalc1).^2)/sum((y - mean(y)).^2)
```

```
Rsqr1 =  
0.822235650485566
```

```
Rsqr2 = 1 - sum((y - yCalc2).^2)/sum((y - mean(y)).^2)
```

```
Rsqr2 =  
0.838210531103428
```

## Residuals and Goodness of Fit

Residuals are the difference between the *observed* values of the response (dependent) variable and the values that a model *predicts*. When you fit a model that is appropriate for your data, the residuals approximate independent random errors. That is, the distribution of residuals ought not to exhibit a discernible pattern.

Producing a fit using a linear model requires minimizing the sum of the squares of the residuals. This minimization yields what is called a least-squares fit. You can gain insight into the “goodness” of a fit by visually examining a plot of the residuals. If the residual plot has a pattern (that is, residual data points do not appear to have a random scatter), the randomness indicates that the model does not properly fit the data.

Evaluate each fit you make in the context of your data. For example, if your goal of fitting the data is to extract coefficients that have physical meaning, then it is important that your model reflect the physics of the data. Understanding what your data represents, how it was measured, and how it is modeled is important when evaluating the goodness of fit.

One measure of goodness of fit is the coefficient of determination, or  $R^2$  (pronounced r-square). This statistic indicates how closely values you obtain from fitting a model match the dependent variable the model is intended to predict. Statisticians often define  $R^2$  using the residual variance from a fitted model:

$$R^2 = 1 - SS_{\text{resid}} / SS_{\text{total}}$$

$SS_{\text{resid}}$  is the sum of the squared residuals from the regression.  $SS_{\text{total}}$  is the sum of the squared differences from the mean of the dependent variable (*total sum of squares*). Both are positive scalars.

To learn how to compute  $R^2$  when you use the Basic Fitting tool, see “Derive  $R^2$ , the Coefficient of Determination” on page 3-24. To learn more about calculating the  $R^2$  statistic and its multivariate generalization, continue reading here.

#### **Example: Computing $R^2$ from Polynomial Fits**

You can derive  $R^2$  from the coefficients of a polynomial regression to determine how much variance in  $y$  a linear model explains, as the following example describes:

- 1 Create two variables,  $x$  and  $y$ , from the first two columns of the count variable in the data file `count.dat`:

```
load count.dat
x = count(:,1);
y = count(:,2);
```

- 2 Use `polyfit` to compute a linear regression that predicts  $y$  from  $x$ :

```
p = polyfit(x,y,1)

p =
    1.5229   -2.1911
```

`p(1)` is the slope and `p(2)` is the intercept of the linear predictor. You can also obtain regression coefficients using the Basic Fitting UI on page 3-16.

- 3 Call `polyval` to use `p` to predict  $y$ , calling the result `yfit`:

```
yfit = polyval(p,x);
```

Using `polyval` saves you from typing the fit equation yourself, which in this case looks like:

```
yfit = p(1) * x + p(2);
```



- 4 Compute the residual values as a vector of signed numbers:

```
yresid = y - yfit;
```

- 5 Square the residuals and total them to obtain the residual sum of squares:

```
SSresid = sum(yresid.^2);
```

- 6 Compute the total sum of squares of  $y$  by multiplying the variance of  $y$  by the number of observations minus 1:

```
SStotal = (length(y)-1) * var(y);
```

- 7 Compute  $R^2$  using the formula given in the introduction of this topic:

```
rsq = 1 - SSresid/SStotal
```

```
rsq =  
0.8707
```

This demonstrates that the linear equation  $1.5229 * x - 2.1911$  predicts 87% of the variance in the variable  $y$ .

### Computing Adjusted R2 for Polynomial Regressions

You can usually reduce the residuals in a model by fitting a higher degree polynomial. When you add more terms, you increase the coefficient of determination,  $R^2$ . You get a closer fit to the data, but at the expense of a more complex model, for which  $R^2$  cannot account. However, a refinement of this statistic, adjusted  $R^2$ , does include a penalty for the number of terms in a model. Adjusted  $R^2$ , therefore, is more appropriate for comparing how different models fit to the same data. The adjusted  $R^2$  is defined as:

$$R^2_{\text{adjusted}} = 1 - (SS_{\text{resid}} / SS_{\text{total}}) * ((n-1)/(n-d-1))$$

where  $n$  is the number of observations in your data, and  $d$  is the degree of the polynomial. (A linear fit has a degree of 1, a quadratic fit 2, a cubic fit 3, and so on.)

The following example repeats the steps of the previous example, “Example: Computing R2 from Polynomial Fits” on page 3-12, but performs a cubic (degree 3) fit instead of a linear (degree 1) fit. From the cubic fit, you compute both simple and adjusted  $R^2$  values to evaluate whether the extra terms improve predictive power:

- 1 Create two variables,  $x$  and  $y$ , from the first two columns of the count variable in the data file `count.dat`:

```
load count.dat
x = count(:,1);
y = count(:,2);
```

- 2** Call `polyfit` to generate a cubic fit to predict `y` from `x`:

```
p = polyfit(x,y,3)
```

```
p =
   -0.0003    0.0390    0.2233    6.2779
```

`p(4)` is the intercept of the cubic predictor. You can also obtain regression coefficients using the Basic Fitting UI on page 3-16.

- 3** Call `polyval` to use the coefficients in `p` to predict `y`, naming the result `yfit`:

```
yfit = polyval(p,x);
```

`polyval` evaluates the explicit equation you could manually enter as:

```
yfit = p(1) * x.^3 + p(2) * x.^2 + p(3) * x + p(4);
```

- 4** Compute the residual values as a vector of signed numbers:

```
yresid = y - yfit;
```

- 5** Square the residuals and total them to obtain the residual sum of squares:

```
SSresid = sum(yresid.^2);
```

- 6** Compute the total sum of squares of `y` by multiplying the variance of `y` by the number of observations minus 1:

```
SStotal = (length(y)-1) * var(y);
```

- 7** Compute simple  $R^2$  for the cubic fit using the formula given in the introduction of this topic:

```
rsq = 1 - SSresid/SStotal
```

```
rsq =
    0.9083
```

- 8** Finally, compute adjusted  $R^2$  to account for degrees of freedom:

```
rsq_adj = 1 - SSresid/SStotal * (length(y)-1)/(length(y)-length(p))
```

```
rsq_adj =
    0.8945
```

The adjusted  $R^2$ , 0.8945, is smaller than simple  $R^2$ , .9083. It provides a more reliable estimate of the power of your polynomial model to predict.

In many polynomial regression models, adding terms to the equation increases both  $R^2$  and adjusted  $R^2$ . In the preceding example, using a cubic fit increased both statistics compared to a linear fit. (You can compute adjusted  $R^2$  for the linear fit for yourself to demonstrate that it has a lower value.) However, it is not always true that a linear fit is worse than a higher-order fit: a more complicated fit can have a lower adjusted  $R^2$  than a simpler fit, indicating that the increased complexity is not justified. Also, while  $R^2$  always varies between 0 and 1 for the polynomial regression models that the Basic Fitting tool generates, adjusted  $R^2$  for some models can be negative, indicating that a model that has too many terms.

Correlation does not imply causality. Always interpret coefficients of correlation and determination cautiously. The coefficients only quantify how much variance in a dependent variable a fitted model removes. Such measures do not describe how appropriate your model—or the independent variables you select—are for explaining the behavior of the variable the model predicts.

## Fitting Data with Curve Fitting Toolbox Functions

The Curve Fitting Toolbox software extends core MATLAB functionality by enabling the following data-fitting capabilities:

- Linear and nonlinear parametric fitting, including standard linear least squares, nonlinear least squares, weighted least squares, constrained least squares, and robust fitting procedures
- Nonparametric fitting
- Statistics for determining the goodness of fit
- Extrapolation, differentiation, and integration
- Dialog box that facilitates data sectioning and smoothing
- Saving fit results in various formats, including MATLAB code files, MAT-files, and workspace variables

For more information, see the Curve Fitting Toolbox documentation.

## Interactive Fitting

In this section...
"The Basic Fitting UI" on page 3-16
"Preparing for Basic Fitting" on page 3-16
"Opening the Basic Fitting UI" on page 3-17
"Example: Using Basic Fitting UI" on page 3-18

### The Basic Fitting UI

The MATLAB Basic Fitting UI allows you to interactively:

- Model data using a spline interpolant, a shape-preserving interpolant, or a polynomial up to the tenth degree
- Plot one or more fits together with data
- Plot the residuals of the fits
- Compute model coefficients
- Compute the norm of the residuals (a statistic you can use to analyze how well a model fits your data)
- Use the model to interpolate or extrapolate outside of the data
- Save coefficients and computed values to the MATLAB workspace for use outside of the dialog box
- Generate MATLAB code to recompute fits and reproduce plots with new data

---

**Note** The Basic Fitting UI is only available for 2-D plots. For more advanced fitting and regression analysis, see the Curve Fitting Toolbox documentation and the Statistics and Machine Learning Toolbox documentation.

---

### Preparing for Basic Fitting

The Basic Fitting UI sorts your data in ascending order before fitting. If your data set is large and the values are not sorted in ascending order, it will take longer for the Basic Fitting UI to preprocess your data before fitting.


You can speed up the Basic Fitting UI by first sorting your data. To create sorted vectors `x_sorted` and `y_sorted` from data vectors `x` and `y`, use the MATLAB `sort` function:

```
[x_sorted, i] = sort(x);  
y_sorted = y(i);
```

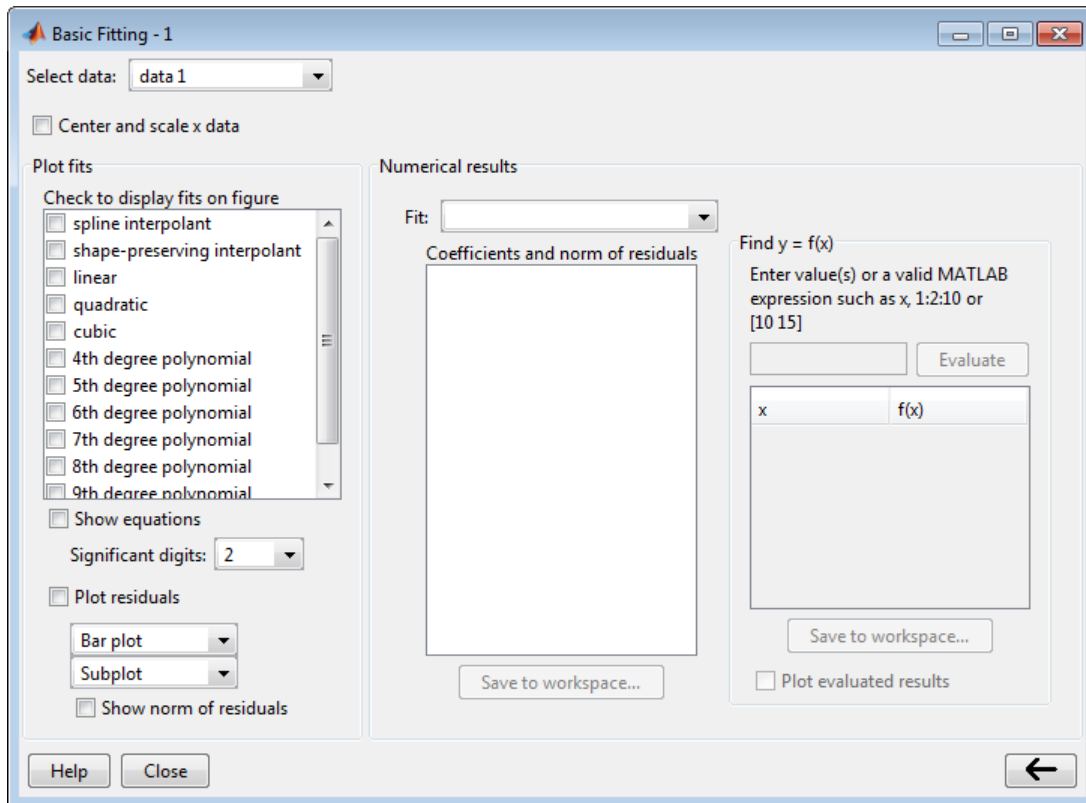
## Opening the Basic Fitting UI

To use the Basic Fitting UI, you must first plot your data in a figure window, using any MATLAB plotting command that produces (only) `x` and `y` data.

To open the Basic Fitting UI, select **Tools > Basic Fitting** from the menus at the top of the figure window.

When you fully expand it by twice clicking the arrow button  in the lower right corner, the window displays three panels. Use these panels to:

- Select a model and plotting options
- Examine and export model coefficients and norms of residuals
- Examine and export interpolated and extrapolated values.



To expand or collapse panels one-by-one, click the arrow button in the lower right corner of the interface.

### Example: Using Basic Fitting UI

This example shows how to use the Basic Fitting UI to fit, visualize, analyze, save, and generate code for polynomial regressions.

- “Load and Plot Census Data” on page 3-19
- “Predict the Census Data with a Cubic Polynomial Fit” on page 3-19
- “View and Save the Cubic Fit Parameters” on page 3-23
- “Derive R2, the Coefficient of Determination” on page 3-24

- “Interpolate and Extrapolate Population Values” on page 3-28
- “Generate a Code File to Reproduce the Result” on page 3-31
- “Learn How the Basic Fitting Tool Computes Fits” on page 3-32

### Load and Plot Census Data

The file, `census.mat`, contains U.S. population data for the years 1790 through 1990 at 10 year intervals.

To load and plot the data, type the following commands at the MATLAB prompt:

```
load census
plot(cdate,pop,'ro')
```

The `load` command adds the following variables to the MATLAB workspace:

- `cdate` — A column vector containing the years from 1790 to 1990 in increments of 10. It is the predictor variable.
- `pop` — A column vector with U.S. population for each year in `cdate`. It is the response variable.

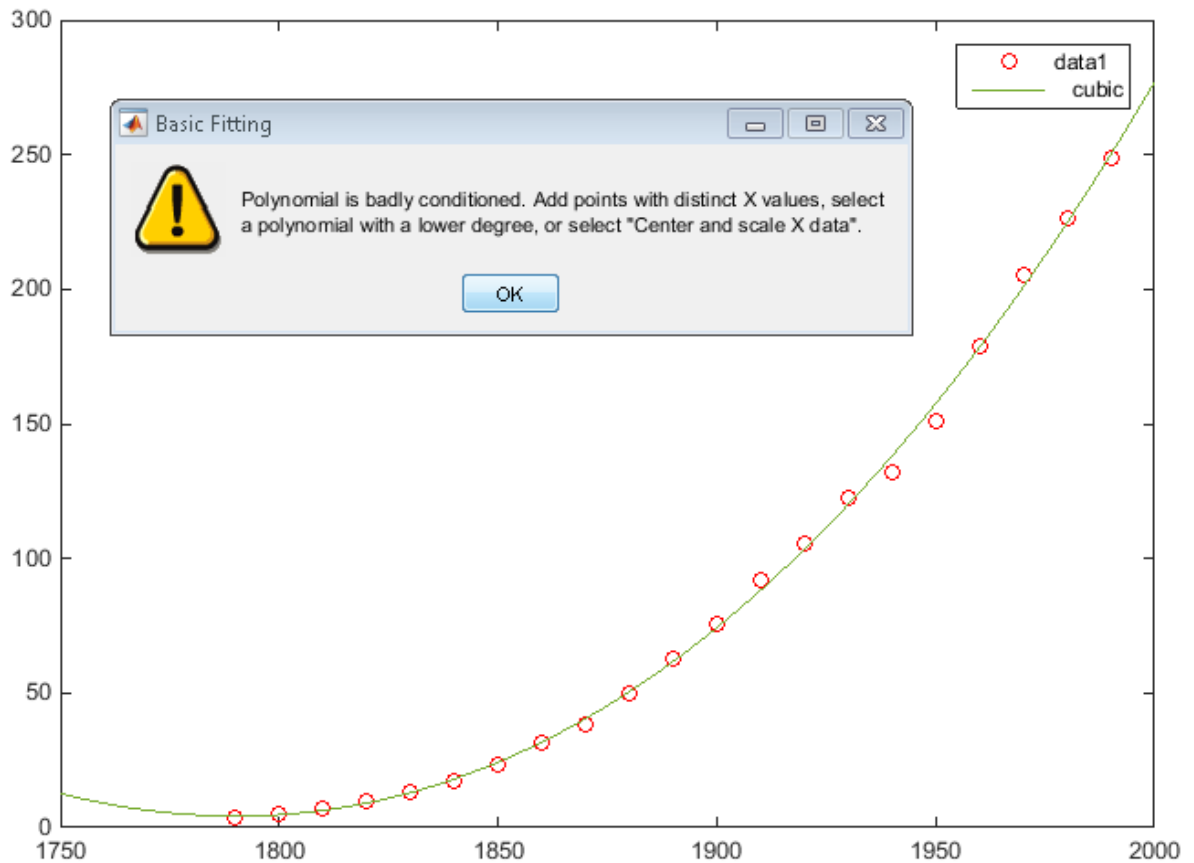
The data vectors are sorted in ascending order, by year. The plot shows the population as a function of year.

Now you are ready to fit an equation the data to model population growth over time.

### Predict the Census Data with a Cubic Polynomial Fit

- 1 Open the Basic Fitting dialog box by selecting **Tools > Basic Fitting** in the Figure window.
- 2 In the **Plot fits** area of the Basic Fitting dialog box, select the **cubic** check box to fit a cubic polynomial to the data.

MATLAB uses your selection to fit the data, and adds the cubic regression line to the graph as follows.



In computing the fit, MATLAB encounters problems and issues the following warning:

```
Polynomial is badly conditioned.  
Add points with distinct X values,  
select a polynomial with a lower degree,  
or select "Center and scale X data."
```

This warning indicates that the computed coefficients for the model are sensitive to random errors in the response (the measured population). It also suggests some things you can do to get a better fit.

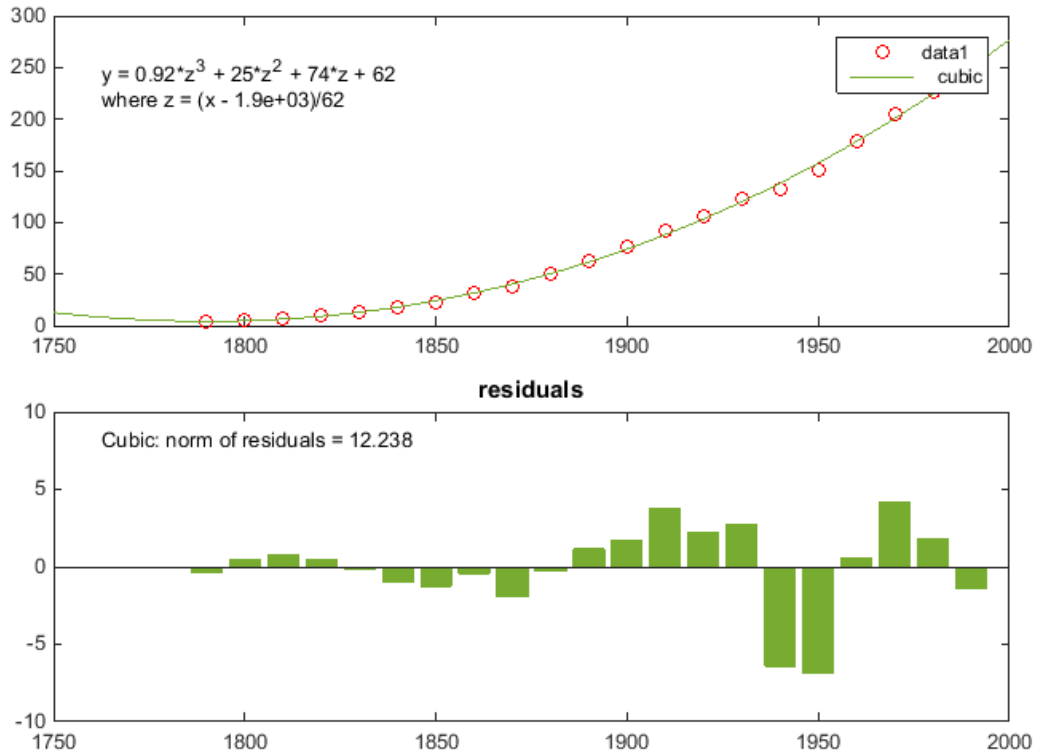


- 3** Continue to use a cubic fit. As you cannot add new observations to the census data, improve the fit by transforming the values you have to *z-scores* before recomputing a fit. Select the **Center and scale X data** check box in the dialog box to make the Basic Fitting tool perform the transformation.

To learn how centering and scaling data works, see “Learn How the Basic Fitting Tool Computes Fits” on page 3-32.

- 4** Now view the equations and display residuals. In addition to selecting the **Center and scale X data** and **cubic** check boxes, select the following options:
- **Show equations**
  - **Plot residuals**
  - **Show norm of residuals**

Selecting **Plot residuals** creates a subplot of them as a bar graph. The following figure displays the results of the Basic Fitting UI options you selected.



The cubic fit is a poor predictor before the year 1790, where it indicates a decreasing population. The model seems to approximate the data reasonably well after 1790. However, a pattern in the residuals shows that the model does not meet the assumption of normal error, which is a basis for the least-squares fitting. The **data 1** line identified in the legend are the observed  $x$  (cdate) and  $y$  (pop) data values. The **cubic** regression line presents the fit after centering and scaling data values. Notice that the figure shows the original data units, even though the tool computes the fit using transformed  $z$ -scores.


For comparison, try fitting another polynomial equation to the census data by selecting it in the **Plot fits** area.

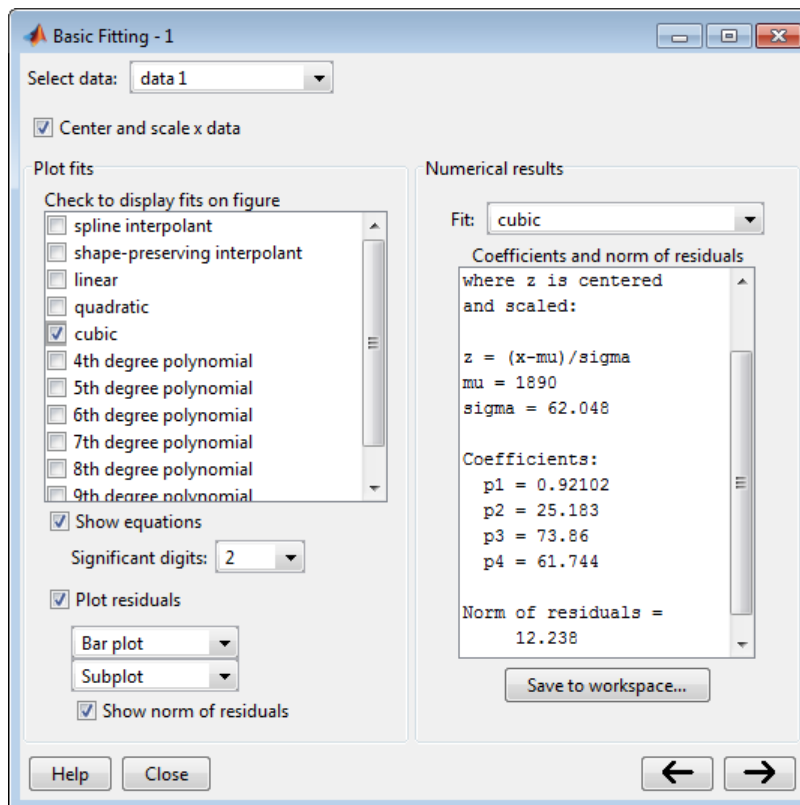
---

**Tip** You can change the default plot settings and rename data series. For more information, see “Customize Graph Using Plot Tools”.

---

### View and Save the Cubic Fit Parameters

In the Basic Fitting dialog box, click the arrow button  to display the estimated coefficients and the norm of the residuals in the **Numerical results** panel.



To view a specific fit, select it from the **Fit** list. This displays the coefficients in the Basic Fitting dialog box, but does not plot the fit in the figure window.

---

**Note** If you also want to display a fit on the plot, you must select the corresponding **Plot fits** check box.

---

Save the fit data to the MATLAB workspace by clicking the **Save to workspace** button on the Numerical results panel. The Save Fit to Workspace dialog box opens.

With all check boxes selected, click **OK** to save the fit parameters as a MATLAB structure:

```
fit
fit =
    type: 'polynomial degree 3'
    coeff: [0.9210 25.1834 73.8598 61.7444]
```

Now, you can use the fit results in MATLAB programming, outside of the Basic Fitting UI.

#### Derive R<sup>2</sup>, the Coefficient of Determination

You can get an indication of how well a polynomial regression predicts your observed data by computing the coefficient of determination, or R-square (written as R<sup>2</sup>). The R<sup>2</sup> statistic, which ranges from 0 to 1, measures how useful the independent variable is in predicting values of the dependent variable:

- An R<sup>2</sup> value near 0 indicates that the fit is not much better than the model  $y = \text{constant}$ .
- An R<sup>2</sup> value near 1 indicates that the independent variable explains most of the variability in the dependent variable.


To compute R<sup>2</sup>, first compute a fit, and then obtain residuals from it. A residual is the signed difference between an observed dependent value and the value your fit predicts for it.

$$\text{residuals} = y_{\text{observed}} - Y_{\text{fitted}}$$

The Basic Fitting tool can generate residuals for any fit it calculates. To view a graph of residuals, select the **Plot residuals** check box. You can view residuals as a bar, line or scatter plot.

After you have residual values, you can save them to the workspace, where you can compute R<sup>2</sup>. Complete the preceding part of this example to fit a cubic polynomial to the census data, and then perform these steps:

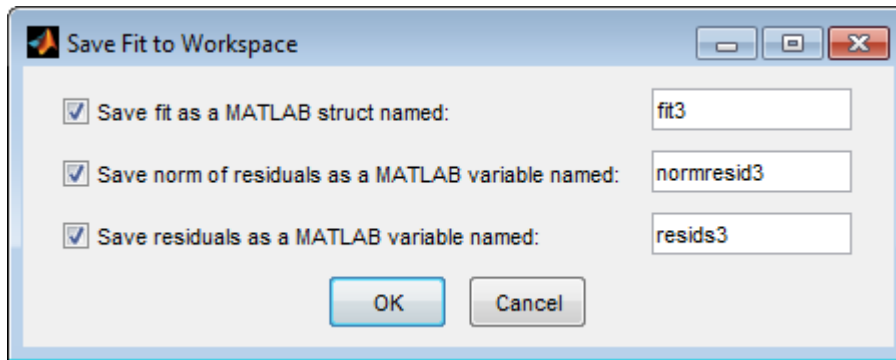
#### Compute Residual Data and R<sup>2</sup> for a Cubic Fit

- 1 Click the arrow button  at the lower right to open the Numerical results tab if it is not already visible.

- 2 From the **Fit** drop-down menu, select cubic if it does not already show.
- 3 Save the fit coefficients, norm of residuals, and residuals by clicking **Save to Workspace**.

The Save Fit to Workspace dialog box opens with three check boxes and three text fields.

- 4 Select all three check boxes to save the fit coefficients, norm of residuals, and residual values.
- 5 Identify the saved variables as belonging to a cubic fit. Change the variable names by adding a 3 to each default name (for example, `fit3`, `normresid3`, and `resids3`). The dialog box should look like this figure.



- 6 Click **OK**. Basic Fitting saves residuals as a column vector of numbers, fit coefficients as a struct, and the norm of residuals as a scalar.

Notice that the value that Basic Fitting computes for norm of residuals is 12.2380. This number is the square root of the sum of squared residuals of the cubic fit.

- 7 Optionally, you can verify the norm-of-residuals value that the Basic Fitting tool provided. Compute the norm-of-residuals yourself from the `resids3` array that you just saved:

```
mynormresid3 = sum(resids3.^2)^(1/2)
```

```
mynormresid3 =  
12.2380
```

- 8 Compute the total sum of squares of the dependent variable, `pop` to compute  $R^2$ . Total sum of squares is the sum of the squared differences of each value from the mean of the variable. For example, use this code:

```
SSpop = (length(pop)-1) * var(pop)
```

```
SSpop =  
1.2356e+005
```

`var(pop)` computes the variance of the population vector. You multiply it by the number of observations after subtracting 1 to account for degrees of freedom. Both the total sum of squares and the norm of residuals are positive scalars.

- 9** Now, compute  $R^2$ , using the square of `normresid3` and `SSpop`:

```
rsqcubic = 1 - normresid3^2 / SSpop
```

```
rsqcubic =  
0.9988
```

- 10** Finally, compute  $R^2$  for a linear fit and compare it with the cubic  $R^2$  value that you just derived. The Basic Fitting UI also provides you with the linear fit results. To obtain the linear results, repeat steps 2-6, modifying your actions as follows:

- To calculate least-squares linear regression coefficients and statistics, in the **Fit** drop-down on the Numerical results pane, select **linear** instead of **cubic**.
- In the Save to Workspace dialog, append **1** to each variable name to identify it as deriving from a linear fit, and click **OK**. The variables `fit1`, `normresid1`, and `resid1` now exist in the workspace.
- Use the variable `normresid1` (98.778) to compute  $R^2$  for the linear fit, as you did in step 9 for the cubic fit:

```
rsqlinear = 1 - normresid1^2 / SSpop
```

```
rsqlinear =  
0.9210
```

This result indicates that a linear least-squares fit of the population data explains 92.1% of its variance. As the cubic fit of this data explains 99.9% of that variance, the latter seems to be a better predictor. However, because a cubic fit predicts using three variables ( $x$ ,  $x^2$ , and  $x^3$ ), a basic  $R^2$  value does not fully reflect how robust the fit is. A more appropriate measure for evaluating the goodness of multivariate fits is adjusted  $R^2$ . For information about computing and using adjusted  $R^2$ , see “Residuals and Goodness of Fit” on page 3-11.

---


**Caution**  $R^2$  measures how well your polynomial equation *predicts* the dependent variable, not how *appropriate* the polynomial model is for your data. When you analyze

inherently unpredictable data, a small value of  $R^2$  indicates that the independent variable does not predict the dependent variable precisely. However, it does not necessarily mean that there is something wrong with the fit.

---

### Compute Residual Data and R2 for a Linear Fit

In this next example, use the Basic Fitting UI to perform a linear fit, save the results to the workspace, and compute  $R^2$  for the linear fit. You can then compare linear  $R^2$  with the cubic  $R^2$  value that you derive in the example “Compute Residual Data and R2 for a Cubic Fit” on page 3-24.

- 1 Click the arrow button  at the lower right to open the Numerical results tab if it is not already visible.
- 2 Select the **linear** check box in the **Plot fits** area.
- 3 From the **Fit** drop-down menu, select **linear** if it does not already show. The Coefficients and norm of residuals area displays statistics for the linear fit.
- 4 Save the fit coefficients, norm of residuals, and residuals by clicking **Save to Workspace**.

The Save Fit to Workspace dialog box opens with three check boxes and three text fields.

- 5 Select all three check boxes to save the fit coefficients, norm of residuals, and residual values.
- 6 Identify the saved variables as belonging to a linear fit. Change the variable names by adding a 1 to each default name (for example, `fit1`, `normresid1`, and `resids1`).
- 7 Click **OK**. Basic Fitting saves residuals as a column vector of numbers, fit coefficients as a struct, and the norm of residuals as a scalar.

Notice that the value that Basic Fitting computes for norm of residuals is 98.778. This number is the square root of the sum of squared residuals of the linear fit.

- 8 Optionally, you can verify the norm-of-residuals value that the Basic Fitting tool provided. Compute the norm-of-residuals yourself from the `resids1` array that you just saved:

```
mynormresid1 = sum(resids1.^2)^(1/2)
```

```
mynormresid1 =  
    98.7783
```

- 9 Compute the total sum of squares of the dependent variable, `pop` to compute  $R^2$ . Total sum of squares is the sum of the squared differences of each value from the mean of the variable. For example, use this code:

```
SSpop = (length(pop)-1) * var(pop)
```

```
SSpop =  
1.2356e+005
```

`var(pop)` computes the variance of the population vector. You multiply it by the number of observations after subtracting 1 to account for degrees of freedom. Both the total sum of squares and the norm of residuals are positive scalars.

- 10 Now, compute  $R^2$ , using the square of `normresid1` and `SSpop`:

```
rsqlinear = 1 - normresid1^2 / SSpop
```

```
rsqcubic =  
0.9210
```

This result indicates that a linear least-squares fit of the population data explains 92.1% of its variance. As the cubic fit of this data explains 99.9% of that variance, the latter seems to be a better predictor. However, a cubic fit has four coefficients ( $x$ ,  $x^2$ ,  $x^3$ , and a constant), while a linear fit has two coefficients ( $x$  and a constant). A simple  $R^2$  statistic does not account for the different degrees of freedom. A more appropriate measure for evaluating polynomial fits is adjusted  $R^2$ . For information about computing and using adjusted  $R^2$ , see “Residuals and Goodness of Fit” on page 3-11.


---

**Caution**  $R^2$  measures how well your polynomial equation *predicts* the dependent variable, not how *appropriate* the polynomial model is for your data. When you analyze inherently unpredictable data, a small value of  $R^2$  indicates that the independent variable does not predict the dependent variable precisely. However, it does not necessarily mean that there is something wrong with the fit.

---

### Interpolate and Extrapolate Population Values

Suppose you want to use the cubic model to interpolate the U.S. population in 1965 (a date not provided in the original data).

- 1 In the Basic Fitting dialog box, click the  button to specify a vector of  $x$  values at which to evaluate the current fit.



- 2 In the **Enter value(s)...** field, type the following value:

1965

**Note** Use unscaled and uncentered x values. You do not need to center and scale first, even though you selected to scale x values to obtain the coefficients in “Predict the Census Data with a Cubic Polynomial Fit” on page 3-19. The Basic Fitting tool makes the necessary adjustments behind the scenes.

- 3 Click **Evaluate**.

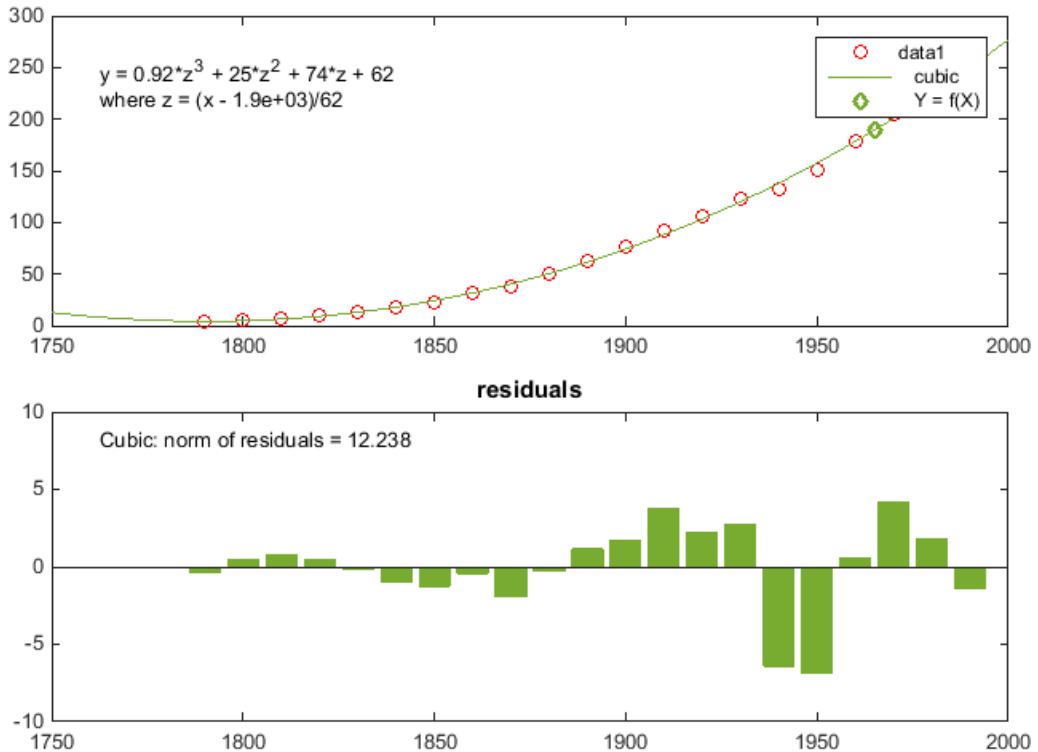
The x values and the corresponding values for  $f(x)$  computed from the fit and displayed in a table, as shown below:

The screenshot shows the 'Basic Fitting - 1' window. The 'Select data' dropdown is set to 'data 1'. The 'Center and scale x data' checkbox is checked. Under 'Plot fits', the 'cubic' option is selected. The 'Numerical results' section shows the fit type as 'cubic' and the equation  $p_3z + p_4$  where  $z = (x - \mu) / \sigma$ . The coefficients are listed as  $p_1 = 0.92102$ ,  $p_2 = 25.183$ ,  $p_3 = 73.86$ , and  $p_4 = 61.744$ . The 'Find y = f(x)' section has the value '1965' entered in the input field. Below this, a table displays the results:

x	f(x)
1.97e+003	189

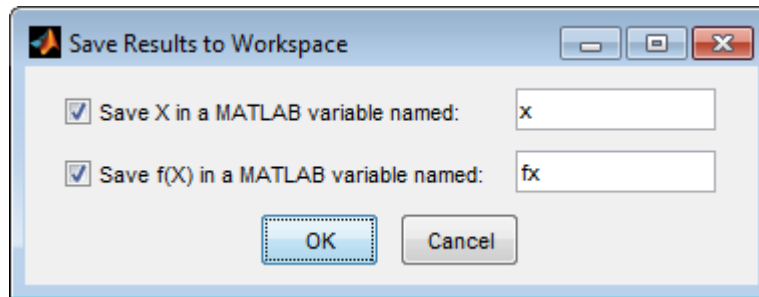
The 'Evaluate' button is visible next to the input field, and the 'Plot evaluated results' checkbox is checked.

- 4 Select the **Plot evaluated results** check box to display the interpolated value as a diamond marker:



- 5 Save the interpolated population in 1965 to the MATLAB workspace by clicking **Save to workspace**.

This opens the following dialog box, where you specify the variable names:



- 6 Click **OK**, but keep the Figure window open if you intend to follow the steps in the next section, “Generate a Code File to Reproduce the Result” on page 3-31.

### Generate a Code File to Reproduce the Result

After completing a Basic Fitting session, you can generate MATLAB code that recomputes fits and reproduces plots with new data.

- 1 In the Figure window, select **File > Generate Code**.

This creates a function and displays it in the MATLAB Editor. The code shows you how to programmatically reproduce what you did interactively with the Basic Fitting dialog box.

- 2 Change the name of the function on the first line from `createfigure` to something more specific, like `censusplot`. Save the code file to your current folder with the file name `censusplot.m`. The function begins with:

```
function censusplot(X1, Y1, valuesToEvaluate1)
```

- 3 Generate some new, randomly perturbed census data:

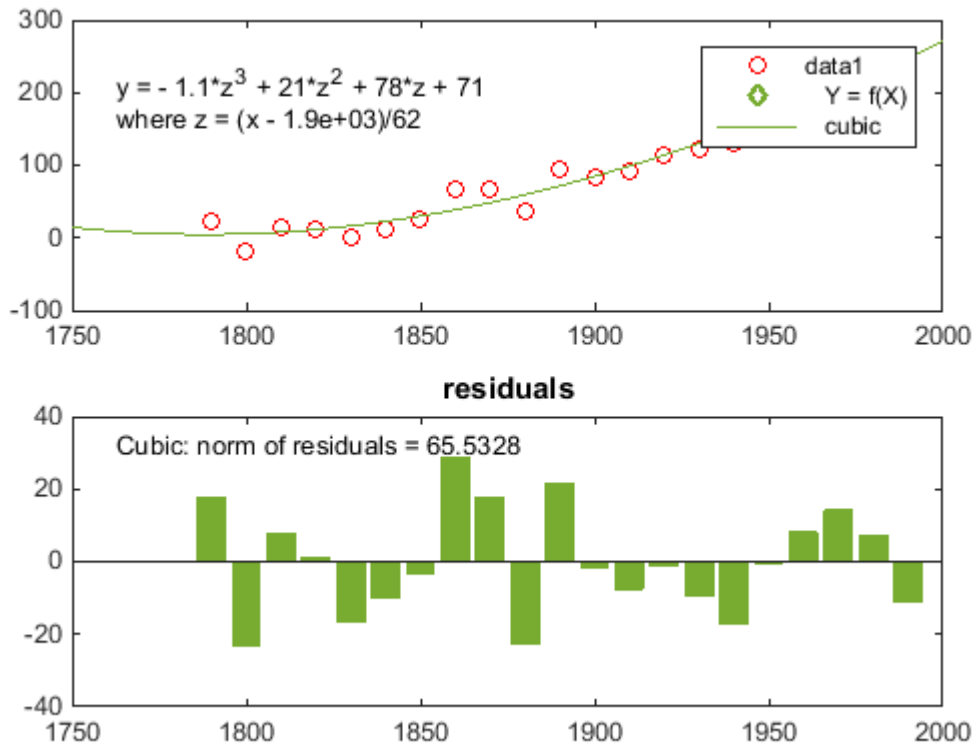
```
randpop = pop + 10*randn(size(pop));
```

- 4 Reproduce the plot with the new data and recompute the fit:

```
censusplot(cdate, randpop, 1965)
```

You need three input arguments:  $x, y$  values (data 1) plotted in the original graph, plus an  $x$ -value for a marker.

The following figure displays the plot that the generated code produces. The new plot matches the appearance of the figure from which you generated code except for the  $y$  data values, the equation for the cubic fit, and the residual values in the bar graph, as expected.



### Learn How the Basic Fitting Tool Computes Fits

The Basic Fitting tool calls the `polyfit` function to compute polynomial fits. It calls the `polyval` function to evaluate the fits. `polyfit` analyzes its inputs to determine if the data is well conditioned for the requested degree of fit.

When it finds badly conditioned data, `polyfit` computes a regression as well as it can, but it also returns a warning that the fit could be improved. The Basic Fitting example section “Predict the Census Data with a Cubic Polynomial Fit” on page 3-19 displays this warning.

One way to improve model reliability is to add data points. However, adding observations to a data set is not always feasible. An alternative strategy is to transform the predictor

variable to normalize its center and scale. (In the example, the predictor is the vector of census dates.)

The `polyfit` function normalizes by computing *z-scores*:

$$z = \frac{x - \mu}{\sigma}$$

where  $x$  is the predictor data,  $\mu$  is the mean of  $x$ , and  $\sigma$  is the standard deviation of  $x$ . The *z-scores* give the data a mean of 0 and a standard deviation of 1. In the Basic Fitting UI, you transform the predictor data to *z-scores* by selecting the **Center and scale x data** check box.

After centering and scaling, model coefficients are computed for the  $y$  data as a function of  $z$ . These are different (and more robust) than the coefficients computed for  $y$  as a function of  $x$ . The form of the model and the norm of the residuals do not change. The Basic Fitting UI automatically rescales the *z-scores* so that the fit plots on the same scale as the original  $x$  data.

To understand the way in which the centered and scaled data is used as an intermediary to create the final plot, run the following code in the Command Window:

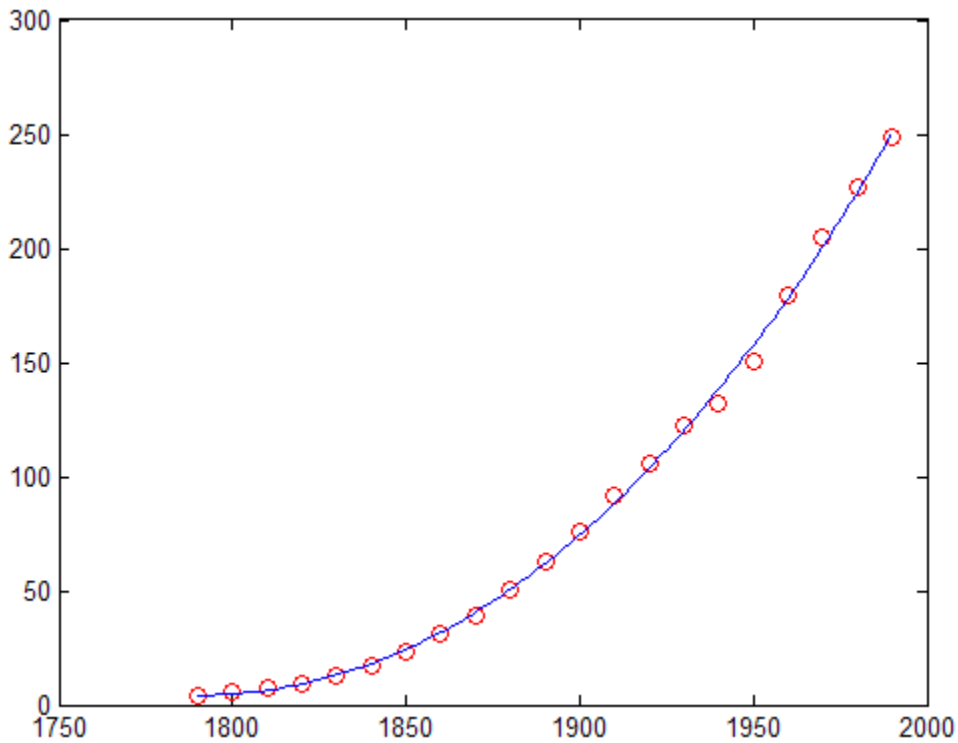
```
close
load census
x = cdate;
y = pop;
z = (x-mean(x))/std(x); % Compute z-scores of x data

plot(x,y,'ro') % Plot data as red markers
hold on % Prepare axes to accept new graph on top

zfit = linspace(z(1),z(end),100);
pz = polyfit(z,y,3); % Compute conditioned fit
yfit = polyval(pz,zfit);

xfit = linspace(x(1),x(end),100);
plot(xfit,yfit,'b-') % Plot conditioned fit vs. x data
```

The centered and scaled cubic polynomial plots as a blue line, as shown here:



In the code, computation of `z` illustrates how to normalize data. The `polyfit` function performs the transformation itself if you provide three return arguments when calling it:

```
[p,S,mu] = polyfit(x,y,n)
```

The returned regression parameters, `p`, now are based on normalized `x`. The returned vector, `mu`, contains the mean and standard deviation of `x`. For more information, see the `polyfit` reference page.

## Programmatic Fitting

### In this section...

“MATLAB Functions for Polynomial Models” on page 3-35

“Linear Model with Nonpolynomial Terms” on page 3-41

“Multiple Regression” on page 3-42

“Programmatic Fitting” on page 3-44

## MATLAB Functions for Polynomial Models

Two MATLAB functions can model your data with a polynomial.

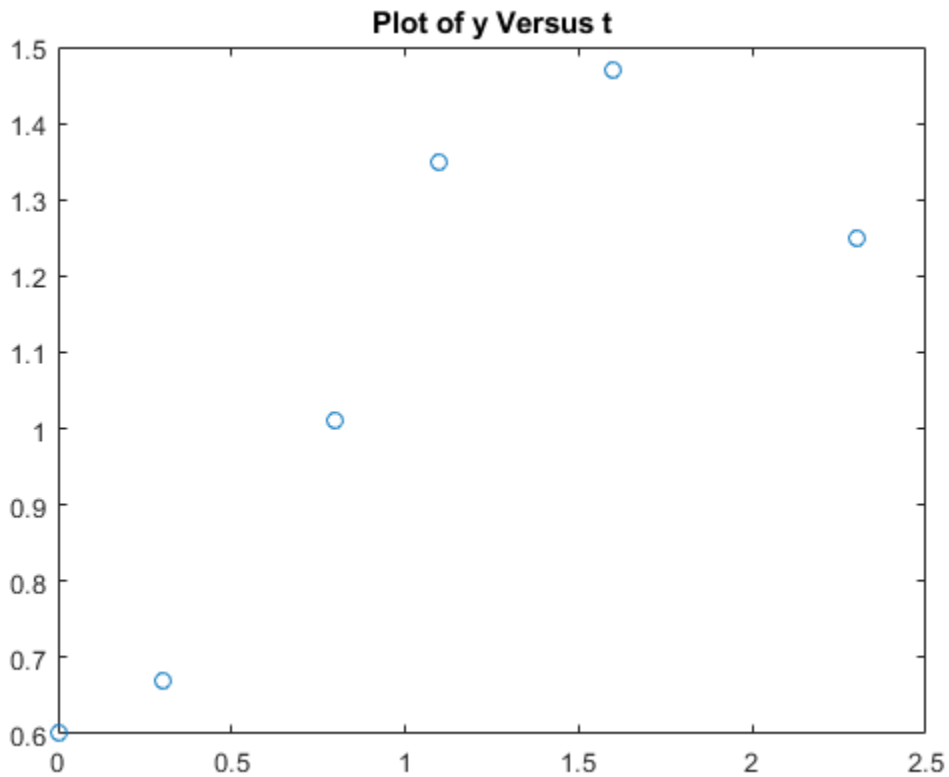
### Polynomial Fit Functions

Function	Description
<code>polyfit</code>	<code>polyfit(x,y,n)</code> finds the coefficients of a polynomial $p(x)$ of degree $n$ that fits the $y$ data by minimizing the sum of the squares of the deviations of the data from the model (least-squares fit).
<code>polyval</code>	<code>polyval(p,x)</code> returns the value of a polynomial of degree $n$ that was determined by <code>polyfit</code> , evaluated at $x$ .

This example shows how to model data with a polynomial.

Measure a quantity  $y$  at several values of time  $t$ .

```
t = [0 0.3 0.8 1.1 1.6 2.3];
y = [0.6 0.67 1.01 1.35 1.47 1.25];
plot(t,y,'o')
title('Plot of y Versus t')
```



You can try modeling this data using a second-degree polynomial function,

$$y = a_2t^2 + a_1t + a_0.$$

The unknown coefficients,  $a_0$ ,  $a_1$ , and  $a_2$ , are computed by minimizing the sum of the squares of the deviations of the data from the model (least-squares fit).

Use `polyfit` to find the polynomial coefficients.

```
p = polyfit(t,y,2)
```

```
p = 1×3
```



```
-0.2942    1.0231    0.4981
```

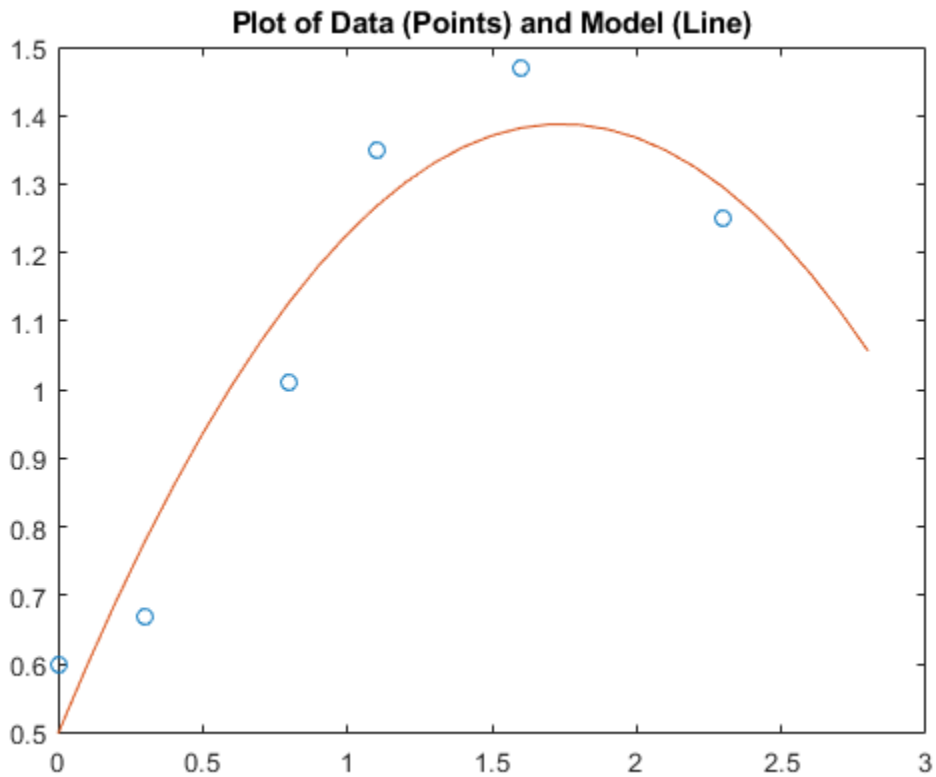
MATLAB calculates the polynomial coefficients in descending powers.

The second-degree polynomial model of the data is given by the equation

$$y = -0.2942t^2 + 1.0231t + 0.4981.$$

Evaluate the polynomial at uniformly spaced times, `t2`. Then, plot the original data and the model on the same plot.

```
t2 = 0:0.1:2.8;  
y2 = polyval(p,t2);  
figure  
plot(t,y,'o',t2,y2)  
title('Plot of Data (Points) and Model (Line)')
```



Evaluate model at the data time vector

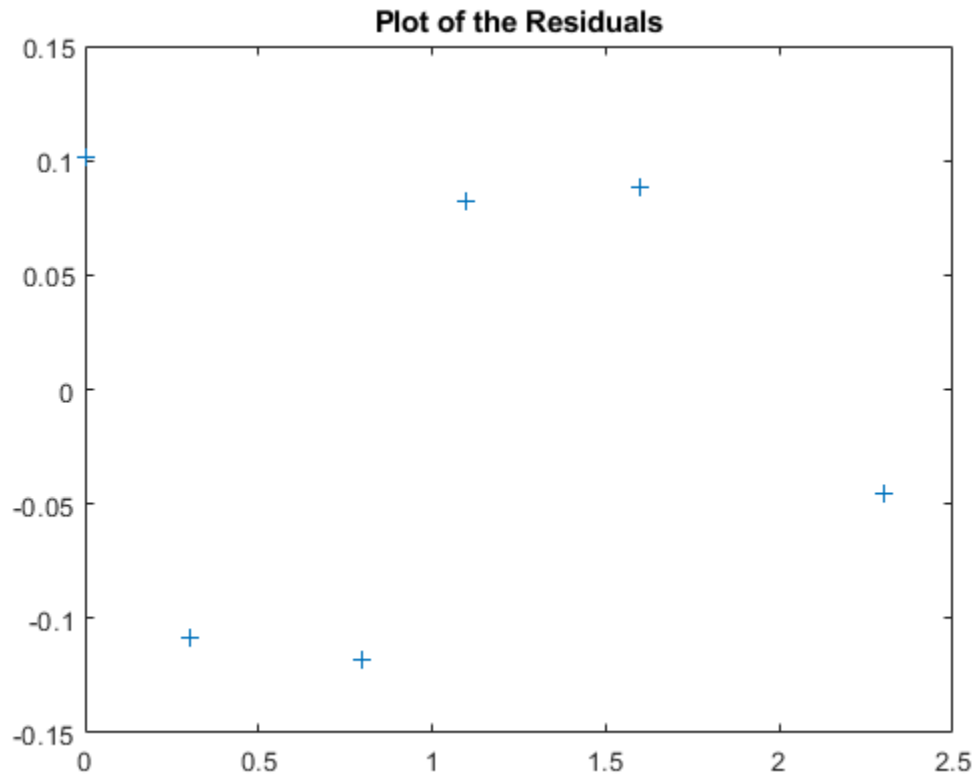
```
y2 = polyval(p,t);
```

Calculate the residuals.

```
res = y - y2;
```

Plot the residuals.

```
figure, plot(t,res,'+')  
title('Plot of the Residuals')
```



Notice that the second-degree fit roughly follows the basic shape of the data, but does not capture the smooth curve on which the data seems to lie. There appears to be a pattern in the residuals, which indicates that a different model might be necessary. A fifth-degree polynomial (shown next) does a better job of following the fluctuations in the data.

Repeat the exercise, this time using a fifth-degree polynomial from `polyfit`.

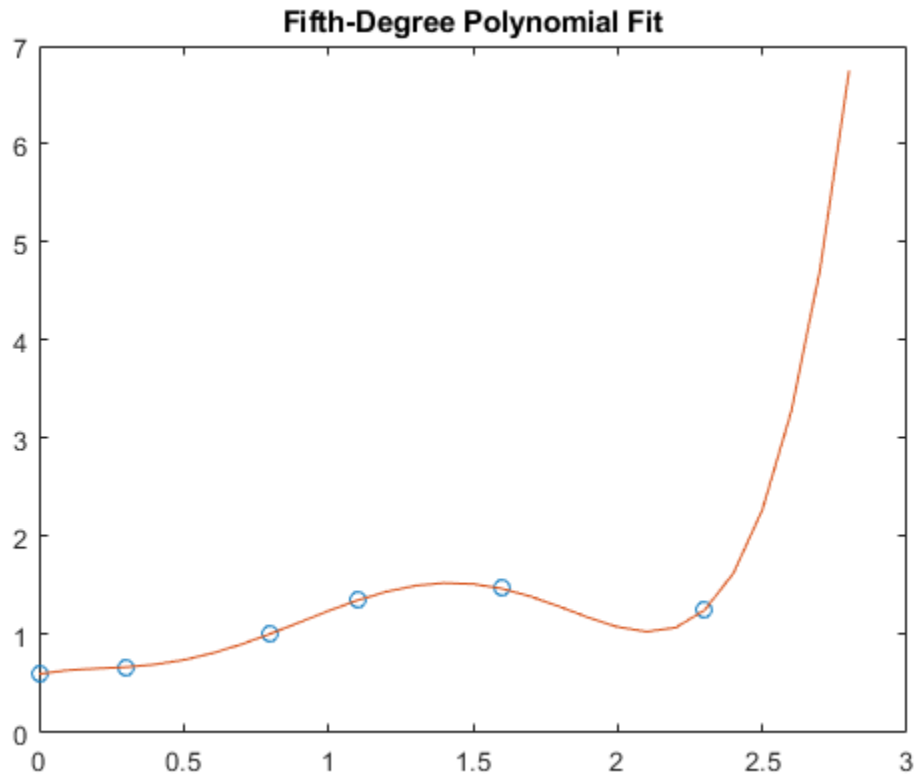
```
p5 = polyfit(t,y,5)
```

```
p5 = 1×6
```

```
0.7303 -3.5892 5.4281 -2.5175 0.5910 0.6000
```

Evaluate the polynomial at  $t_2$  and plot the fit on top of the data in a new figure window.

```
y3 = polyval(p5,t2);  
figure  
plot(t,y,'o',t2,y3)  
title('Fifth-Degree Polynomial Fit')
```



---

**Note** If you are trying to model a physical situation, it is always important to consider whether a model of a specific order is meaningful in your situation.

---

## Linear Model with Nonpolynomial Terms

This example shows how to fit data with a linear model containing nonpolynomial terms.

When a polynomial function does not produce a satisfactory model of your data, you can try using a linear model with nonpolynomial terms. For example, consider the following function that is linear in the parameters  $a_0$ ,  $a_1$ , and  $a_2$ , but nonlinear in the  $t$  data:

$$y = a_0 + a_1e^{-t} + a_2te^{-t}.$$

You can compute the unknown coefficients  $a_0$ ,  $a_1$ , and  $a_2$  by constructing and solving a set of simultaneous equations and solving for the parameters. The following syntax accomplishes this by forming a *design matrix*, where each column represents a variable used to predict the response (a term in the model) and each row corresponds to one observation of those variables.

Enter  $t$  and  $y$  as column vectors.

```
t = [0 0.3 0.8 1.1 1.6 2.3]';
y = [0.6 0.67 1.01 1.35 1.47 1.25]';
```

Form the design matrix.

```
X = [ones(size(t)) exp(-t) t.*exp(-t)];
```

Calculate model coefficients.

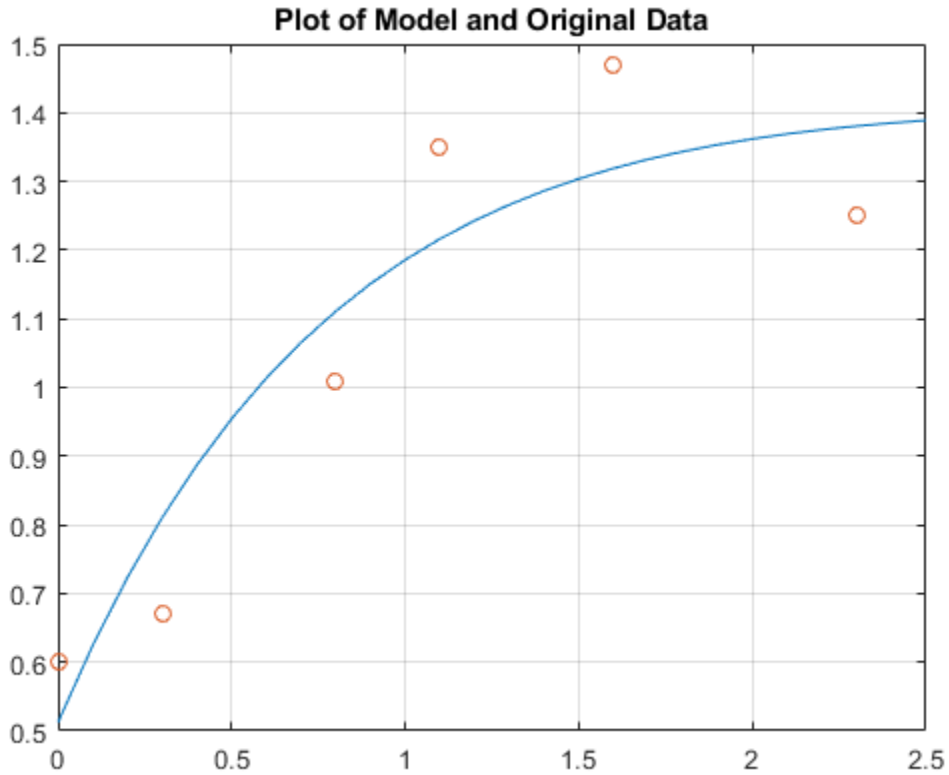
```
a = X\y
a = 3×1
    1.3983
   -0.8860
    0.3085
```

Therefore, the model of the data is given by

$$y = 1.3983 - 0.8860e^{-t} + 0.3085te^{-t}.$$

Now evaluate the model at regularly spaced points and plot the model with the original data.

```
T = (0:0.1:2.5)';  
Y = [ones(size(T)) exp(-T) T.*exp(-T)]*a;  
plot(T,Y,'-',t,y,'o'), grid on  
title('Plot of Model and Original Data')
```



## Multiple Regression

This example shows how to use multiple regression to model data that is a function of more than one predictor variable.

When  $y$  is a function of more than one predictor variable, the matrix equations that express the relationships among the variables must be expanded to accommodate the additional data. This is called *multiple regression*.

Measure a quantity  $y$  for several values of  $x_1$  and  $x_2$ . Store these values in vectors  $x_1$ ,  $x_2$ , and  $y$ , respectively.

```
x1 = [.2 .5 .6 .8 1.0 1.1]';
x2 = [.1 .3 .4 .9 1.1 1.4]';
y  = [.17 .26 .28 .23 .27 .24]';
```

A model of this data is of the form

$$y = a_0 + a_1x_1 + a_2x_2.$$

Multiple regression solves for unknown coefficients  $a_0$ ,  $a_1$ , and  $a_2$  by minimizing the sum of the squares of the deviations of the data from the model (least-squares fit).

Construct and solve the set of simultaneous equations by forming a design matrix,  $X$ .

```
X = [ones(size(x1)) x1 x2];
```

Solve for the parameters by using the backslash operator.

```
a = X\y
```

```
a = 3×1
```

```
    0.1018
    0.4844
   -0.2847
```

The least-squares fit model of the data is

$$y = 0.1018 + 0.4844x_1 - 0.2847x_2.$$

To validate the model, find the maximum of the absolute value of the deviation of the data from the model.

```
Y = X*a;
MaxErr = max(abs(Y - y))
```

```
MaxErr = 0.0038
```

This value is much smaller than any of the data values, indicating that this model accurately follows the data.

### Programmatic Fitting

This example shows how to use MATLAB functions to:

- “Calculate Correlation Coefficients” on page 3-45
- “Fit a Polynomial to the Data” on page 3-46
- “Plot and Calculate Confidence Bounds” on page 3-48

Load sample census data from `census.mat`, which contains U.S. population data from the years 1790 to 1990.

```
load census
```

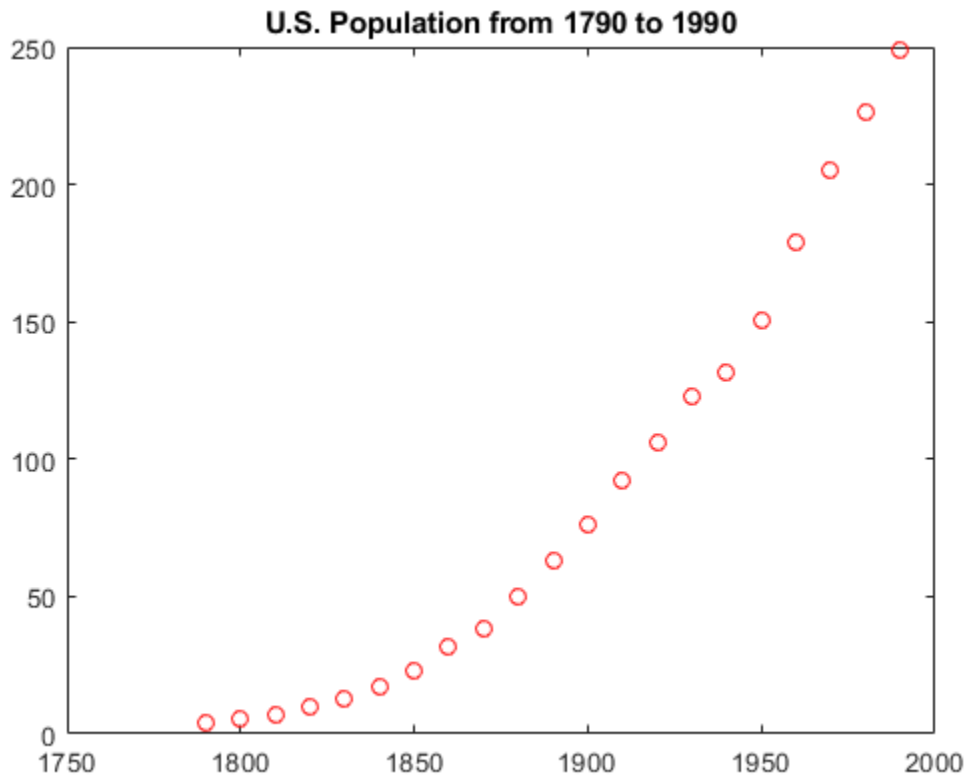
This adds the following two variables to the MATLAB workspace.

- `cdate` is a column vector containing the years 1790 to 1990 in increments of 10.
- `pop` is a column vector with the U.S. population numbers corresponding to each year in `cdate`.

Plot the data.

```
plot(cdate,pop,'ro')  
title('U.S. Population from 1790 to 1990')
```





The plot shows a strong pattern, which indicates a high correlation between the variables.

### Calculate Correlation Coefficients

In this portion of the example, you determine the statistical correlation between the variables `cdate` and `pop` to justify modeling the data. For more information about correlation coefficients, see “Linear Correlation” on page 3-2.

Calculate the correlation-coefficient matrix.

```
corrcoef(cdate,pop)
```

```
ans = 2x2
```

```
1.0000    0.9597
0.9597    1.0000
```

The diagonal matrix elements represent the perfect correlation of each variable with itself and are equal to 1. The off-diagonal elements are very close to 1, indicating that there is a strong statistical correlation between the variables `cdate` and `pop`.

#### **Fit a Polynomial to the Data**

This portion of the example applies the `polyfit` and `polyval` MATLAB functions to model the data.

Calculate fit parameters.

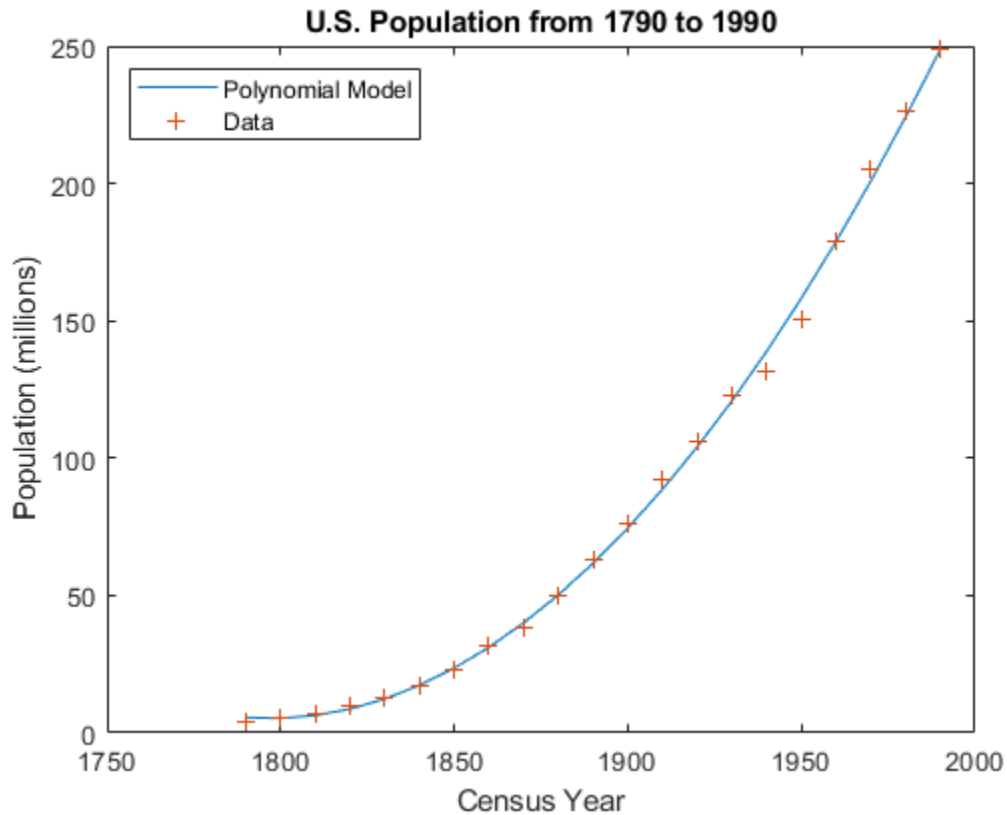
```
[p,ErrorEst] = polyfit(cdate,pop,2);
```

Evaluate the fit.

```
pop_fit = polyval(p,cdate,ErrorEst);
```

Plot the data and the fit.

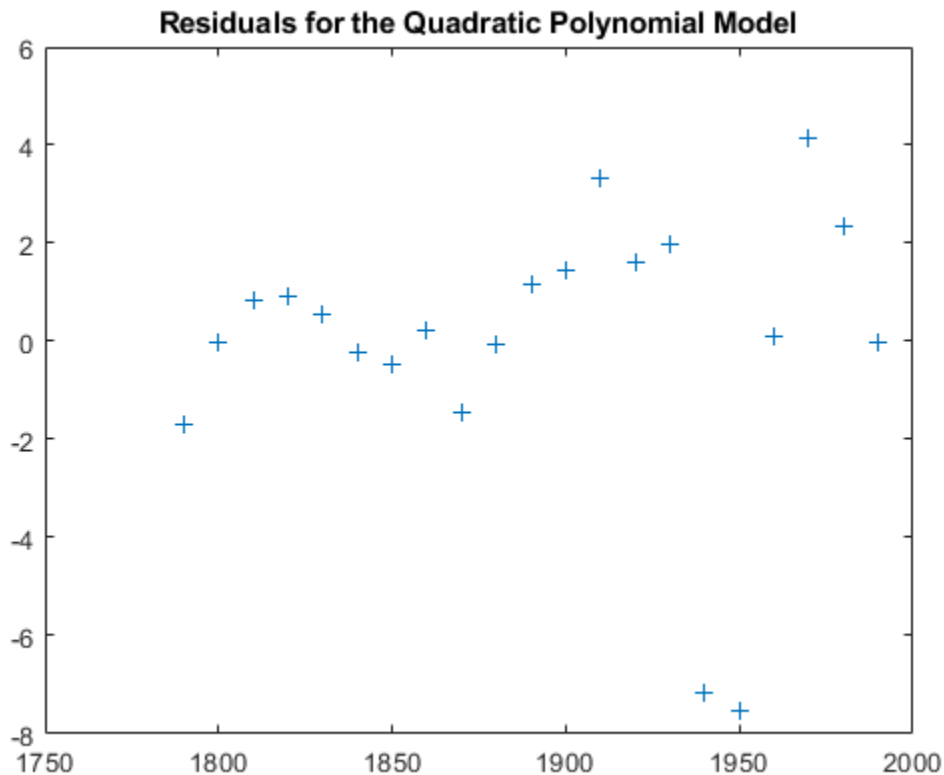
```
plot(cdate,pop_fit,'-',cdate,pop,'+');
title('U.S. Population from 1790 to 1990')
legend('Polynomial Model','Data','Location','NorthWest');
xlabel('Census Year');
ylabel('Population (millions)');
```



The plot shows that the quadratic-polynomial fit provides a good approximation to the data.

Calculate the residuals for this fit.

```
res = pop - pop_fit;
figure, plot(cdate,res,'+')
title('Residuals for the Quadratic Polynomial Model')
```



Notice that the plot of the residuals exhibits a pattern, which indicates that a second-degree polynomial might not be appropriate for modeling this data.

#### **Plot and Calculate Confidence Bounds**

Confidence bounds are confidence intervals for a predicted response. The width of the interval indicates the degree of certainty of the fit.

This portion of the example applies `polyfit` and `polyval` to the census sample data to produce confidence bounds for a second-order polynomial model.

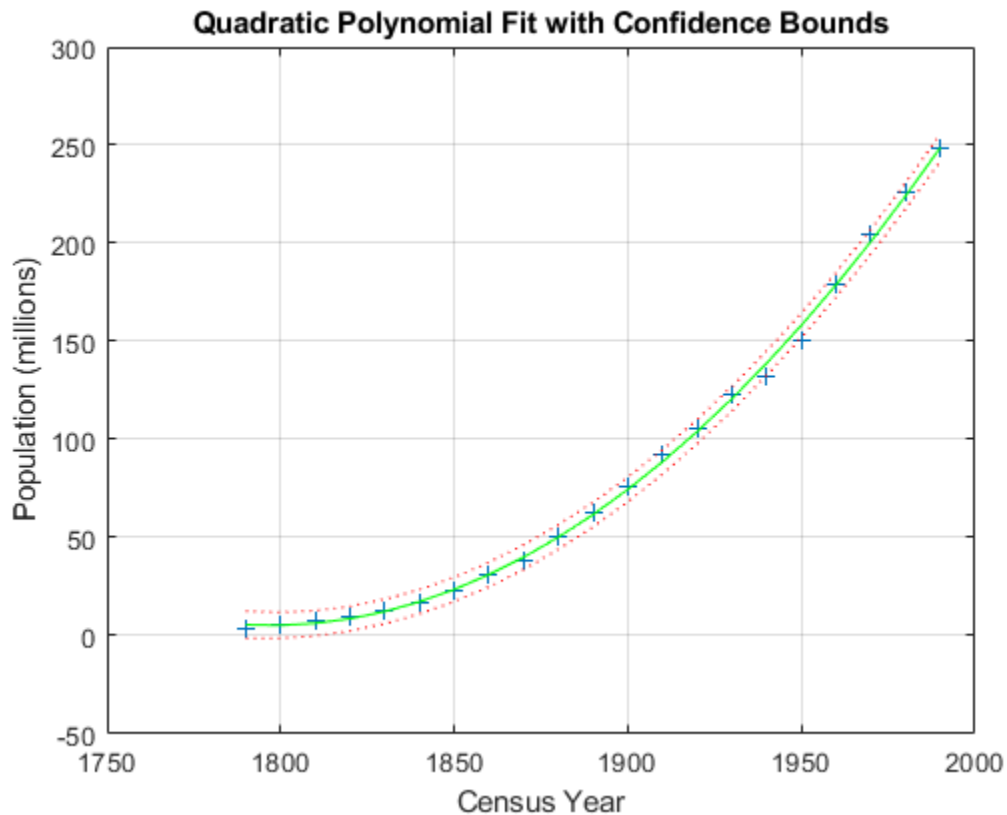
The following code uses an interval of  $\pm 2\Delta$ , which corresponds to a 95% confidence interval for large samples.

Evaluate the fit and the prediction error estimate (delta).

```
[pop_fit,delta] = polyval(p,cdate,ErrorEst);
```

Plot the data, the fit, and the confidence bounds.

```
plot(cdate,pop,'+',...
     cdate,pop_fit,'g-',...
     cdate,pop_fit+2*delta,'r:',...
     cdate,pop_fit-2*delta,'r:');
xlabel('Census Year');
ylabel('Population (millions)');
title('Quadratic Polynomial Fit with Confidence Bounds')
grid on
```



The 95% interval indicates that you have a 95% chance that a new observation will fall within the bounds.

# Time Series Analysis

---

- “What Are Time Series?” on page 4-2
- “Time Series Objects” on page 4-3

# What Are Time Series?

Time series are data vectors sampled over time, in order, often at regular intervals. They are distinguished from randomly sampled data, which form the basis of many other data analyses. Time series represent the time-evolution of a dynamic population or *process*. The linear ordering of time series gives them a distinctive place in data analysis, with a specialized set of techniques.

Time series analysis is concerned with:

- Identifying patterns
- Modeling patterns
- Forecasting values

Several dedicated MATLAB functions perform time series analysis. This section introduces objects and interactive tools for time series analysis.



## Time Series Objects

### In this section...

“Types of Time Series and Their Uses” on page 4-3

“Time Series Data Sample” on page 4-3

“Example: Time Series Objects and Methods” on page 4-5

“Time Series Constructor” on page 4-17

“Time Series Collection Constructor” on page 4-17

### Types of Time Series and Their Uses

MATLAB time series objects are of two types:

- `timeseries` — Stores data and time values, as well as the metadata information that includes units, events, data quality, and interpolation method
- `tscollection` — Stores a collection of `timeseries` objects that share a common time vector, convenient for performing operations on synchronized time series with different units

This section discusses the following topics:

- Using time series constructors to instantiate time series classes
- Modifying object properties using `set` methods or dot notation
- Calling time series functions and methods

To get a quick overview of programming with `timeseries` and `tscollection` objects, follow the steps in “Example: Time Series Objects and Methods” on page 4-5.

### Time Series Data Sample

To properly understand the description of `timeseries` object properties and methods in this documentation, it is important to clarify some terms related to storing data in a `timeseries` object—the difference between a *data value* and a *data sample*.

A *data value* is a single, scalar value recorded at a specific time. A *data sample* consists of one or more values associated with a specific time in the `timeseries` object. The number of data samples in a time series is the same as the length of the time vector.

For example, consider data that consists of three sensor signals: two signals represent the position of an object in meters, and the third represents its velocity in meters/second.

To enter the data matrix, type the following at the MATLAB prompt:

```
x = [-0.2 -0.3 13;  
     -0.1 -0.4 15;  
      NaN  2.8 17;  
      0.5  0.3 NaN;  
     -0.3 -0.1 15]
```

The NaN value represents a missing data value. MATLAB displays the following 5-by-3 matrix:

```
x=  
-0.2000    -0.3000    13.0000  
-0.1000    -0.4000    15.0000  
NaN        2.8000    17.0000  
0.5000     0.3000         NaN  
-0.3000    -0.1000    15.0000
```

The first two columns of `x` contain quantities with the same units and you can create a multivariate `timeseries` object to store these two time series. For more information about creating `timeseries` objects, see “Time Series Constructor” on page 4-17. The following command creates a `timeseries` object `ts_pos` to store the position values:

```
ts_pos = timeseries(x(:,1:2), 1:5, 'name', 'Position')
```

MATLAB responds by displaying the following properties of `ts_pos`:

```
timeseries
```

```
Common Properties:
```

```
    Name: 'Position'  
    Time: [5x1 double]  
TimeInfo: [1x1 tsdata.timemetadata]  
    Data: [5x2 double]  
DataInfo: [1x1 tsdata.datametadata]
```

```
More properties, Methods
```

The `Length` of the time vector, which is 5 in this example, equals the number of data samples in the `timeseries` object. Find the size of the data sample in `ts_pos` by typing the following at the MATLAB prompt:

```
getdatasamplesize(ts_pos)
```

```
ans =
```

```
1     2
```

Similarly, you can create a second `timeseries` object to store the velocity data:

```
ts_vel = timeseries(x(:,3), 1:5, 'name', 'Velocity');
```

Find the size of each data sample in `ts_vel` by typing the following:

```
getdatasamplesize(ts_vel)
```

```
ans =
```

```
1     1
```

Notice that `ts_vel` has one data value in each data sample and `ts_pos` has two data values in each data sample.

---

**Note** In general, when the time series data is an  $M$ -by- $N$ -by- $P$ -by-... multidimensional array with  $M$  samples, the size of each data sample is  $N$ -by- $P$ -by-... .

---

If you want to perform operations on the `ts_pos` and `ts_vel` `timeseries` objects while keeping them synchronized, group them in a time series collection. For more information, see “Time Series Collection Constructor Syntax” on page 4-18.

## Example: Time Series Objects and Methods

- “Creating Time Series Objects” on page 4-6
- “Viewing Time Series Objects” on page 4-7
- “Modifying Time Series Units and Interpolation Method” on page 4-9
- “Defining Events” on page 4-9
- “Creating Time Series Collection Objects” on page 4-10
- “Resampling a Time Series Collection Object” on page 4-11
- “Adding a Data Sample to a Time Series Collection Object” on page 4-12
- “Removing and Interpolating Missing Data” on page 4-13

- “Removing a Time Series from a Time Series Collection” on page 4-15
- “Displaying Time Vector Values as Date Strings” on page 4-15
- “Plotting Time Series Collection Members” on page 4-15

### Creating Time Series Objects

This portion of the example illustrates how to create several `timeseries` objects from an array. For more information about the `timeseries` object, see “Time Series Constructor” on page 4-17.

Import the sample data from `count.dat` to the MATLAB workspace.

```
load count.dat
```

This adds the 24-by-3 matrix, `count`, to the workspace. Each column of `count` represents hourly vehicle counts at each of three town intersections.

View the `count` matrix.

```
count
```

Create three `timeseries` objects to store the data collected at each intersection.

```
count1 = timeseries(count(:,1), 1:24, 'name', 'intersection1');  
count2 = timeseries(count(:,2), 1:24, 'name', 'intersection2');  
count3 = timeseries(count(:,3), 1:24, 'name', 'intersection3');
```

---

**Note** In the above construction, `timeseries` objects have both a variable name (e.g., `count1`) and an internal object name (e.g., `intersection1`). The variable name is used with MATLAB functions. The object name is a property of the object, accessed with object methods. For more information on `timeseries` object properties and methods, see “Time Series Properties” on page 4-17 and “Time Series Methods” on page 4-17.

---

By default, a time series has a time vector having units of seconds and a start time of 0 sec. The example constructs the `count1`, `count2`, and `count3` time series objects with start times of 1 sec, end times of 24 sec, and 1-sec increments. You will change the time units to hours in “Modifying Time Series Units and Interpolation Method” on page 4-9.

---

**Note** If you want to create a `timeseries` object that groups the three data columns in `count`, use the following syntax:

```
count_ts = timeseries(count, 1:24, 'name', 'traffic_counts')
```

This is useful when all time series have the same units and you want to keep them synchronized during calculations.

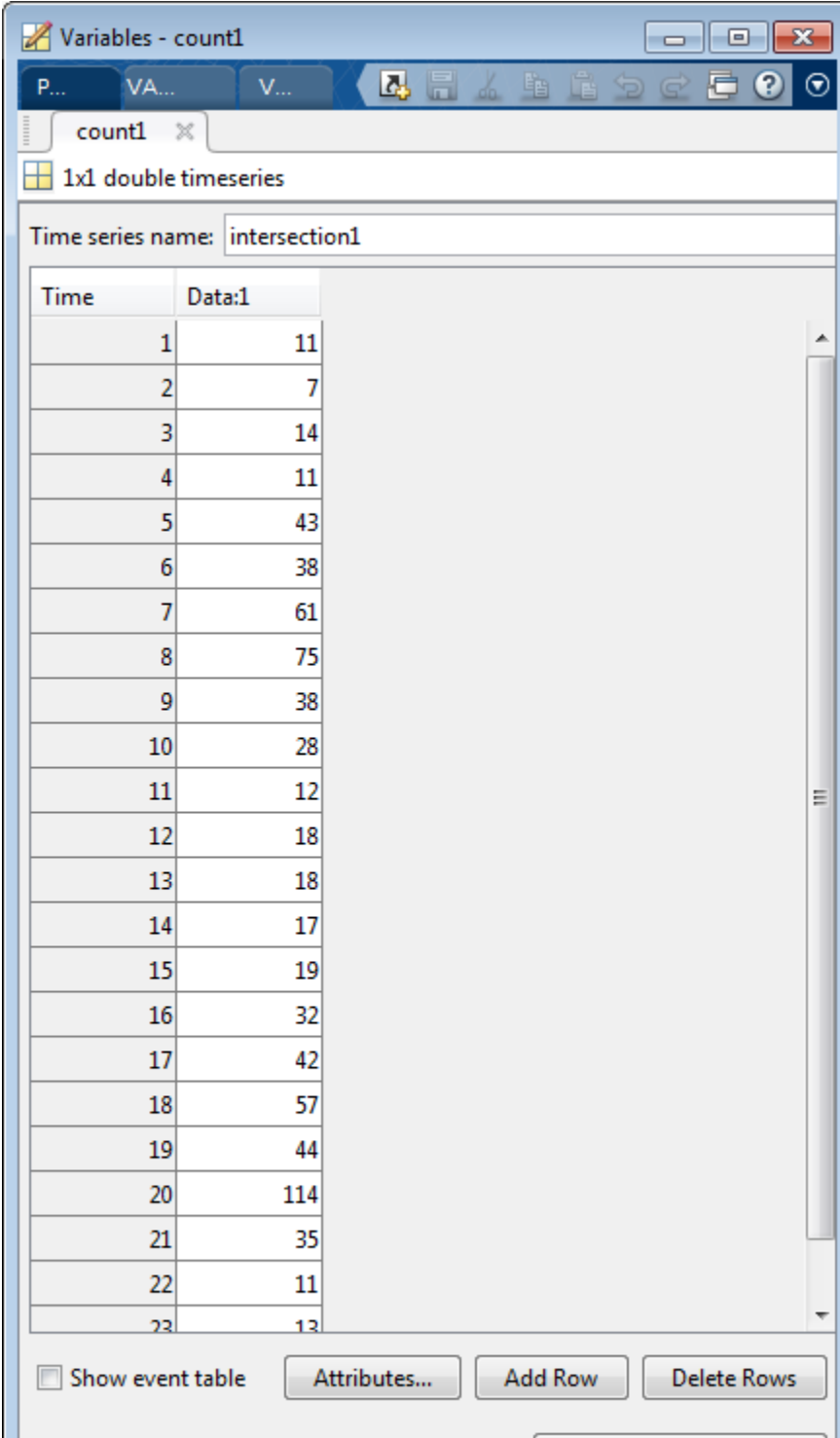
---

### Viewing Time Series Objects

After creating a `timeseries` object, as described in “Creating Time Series Objects” on page 4-6, you can view it in the Variables editor.

To view a `timeseries` object like `count1` in the Variables editor, use either of the following methods:

- Type `open('count1')` at the command prompt.
- On the **Home** tab, in the **Variable** section, click **Open Variable** and select `count1`.



The screenshot shows a software window titled "Variables - count1". The window contains a time series plot for "intersection1". The plot displays a line graph with data points connected by lines. The x-axis is labeled "Time" and ranges from 1 to 23. The y-axis is labeled "Data:1" and ranges from 0 to 114. The data points are as follows:

Time	Data:1
1	11
2	7
3	14
4	11
5	43
6	38
7	61
8	75
9	38
10	28
11	12
12	18
13	18
14	17
15	19
16	32
17	42
18	57
19	44
20	114
21	35
22	11
23	13

At the bottom of the window, there are several controls: a checkbox labeled "Show event table" (which is unchecked), and three buttons: "Attributes...", "Add Row", and "Delete Rows".

## Modifying Time Series Units and Interpolation Method

After creating a `timeseries` object, as described in “Creating Time Series Objects” on page 4-6, you can modify its units and interpolation method using dot notation.

View the current properties of `count1`.

```
get(count1)
```

MATLAB displays the current property values of the `count1` `timeseries` object.

View the current `DataInfo` properties using dot notation.

```
count1.DataInfo
```

Change the data units for `count1` to 'cars'.

```
count1.DataInfo.Units = 'cars';
```

Set the interpolation method for `count1` to zero-order hold.

```
count1.DataInfo.Interpolation = tsdata.interpolation('zoh');
```

Verify that the `DataInfo` properties have been modified.

```
count1.DataInfo
```

Modify the time units to be 'hours' for the three time series.

```
count1.TimeInfo.Units = 'hours';
count2.TimeInfo.Units = 'hours';
count3.TimeInfo.Units = 'hours';
```

## Defining Events

This portion of the example illustrates how to define events for a `timeseries` object by using the `tsdata.event` auxiliary object. Events mark the data at specific times. When you plot the data, event markers are displayed on the plot. Events also provide a convenient way to synchronize multiple time series.

Add two events to the data that mark the times of the AM commute and PM commute.

Construct and add the first event to all time series. The first event occurs at 8 AM.

```
e1 = tsdata.event('AMCommute',8);
e1.Units = 'hours';           % Specify the units for time
```

```
count1 = addevent(count1,e1); % Add the event to count1
count2 = addevent(count2,e1); % Add the event to count2
count3 = addevent(count3,e1); % Add the event to count3
```

Construct and add the second event to all time series. The second event occurs at 6 PM.

```
e2 = tsdata.event('PMCommute',18);
e2.Units = 'hours'; % Specify the units for time
count1 = addevent(count1,e2); % Add the event to count1
count2 = addevent(count2,e2); % Add the event to count2
count3 = addevent(count3,e2); % Add the event to count3
```

Plot the time series, count1.

```
figure
plot(count1)
```

When you plot any of the time series, the plot method defined for time series objects displays events as markers. By default markers are red filled circles.

The plot reflects that count1 uses zero-order-hold interpolation.

Plot count2.

```
plot(count2)
```

If you plot time series count2, it replaces the count1 display. You see its events and that it uses linear interpolation.

Overlay time series plots by setting hold on.

```
hold on
plot(count3)
```

### Creating Time Series Collection Objects

This portion of the example illustrates how to create a `tscollection` object. Each individual time series in a collection is called a *member*. For more information about the `tscollection` object, see “Time Series Collection Constructor” on page 4-17.

---

**Note** Typically, you use the `tscollection` object to group synchronized time series that have different units. In this simple example, all time series have the same units and the `tscollection` object does not provide an advantage over grouping the three time series



in a single `timeseries` object. For an example of how to group several time series in one `timeseries` object, see “Creating Time Series Objects” on page 4-6.

---

Create a `tscollection` object named `count_coll` and use the constructor syntax to immediately add two of the three time series currently in the MATLAB workspace (you will add the third time series later).

```
tsc = tscollection({count1 count2}, 'name', 'count_coll')
```

---

**Note** The time vectors of the `timeseries` objects you are adding to the `tscollection` must match.

---

Notice that the `Name` property of the `timeseries` objects is used to name the collection members as `intersection1` and `intersection2`.

Add the third `timeseries` object in the workspace to the `tscollection`.

```
tsc = addts(tsc, count3)
```

All three members in the collection are listed.

### Resampling a Time Series Collection Object

This portion of the example illustrates how to resample each member in a `tscollection` using a new time vector. The resampling operation is used to either select existing data at specific time values, or to interpolate data at finer intervals. If the new time vector contains time values that did not exist in the previous time vector, the new data values are calculated using the default interpolation method you associated with the time series.

Resample the time series to include data values every 2 hours instead of every hour and save it as a new `tscollection` object.

```
tsc1 = resample(tsc, 1:2:24)
```

In some cases you might need a finer sampling of information than you currently have and it is reasonable to obtain it by interpolating data values.

Interpolate values at each half-hour mark.

```
tsc1 = resample(tsc, 1:0.5:24)
```

To add values at each half-hour mark, the default interpolation method of a time series is used. For example, the new data points in `intersection1` are calculated by using the zero-order hold interpolation method, which holds the value of the previous sample constant. You set the interpolation method for `intersection1` as described in “Modifying Time Series Units and Interpolation Method” on page 4-9.

The new data points in `intersection2` and `intersection3` are calculated using linear interpolation, which is the default method.

Plot the members of `tscl` with markers to see the results of interpolating.

```
hold off % Allow axes to clear before plotting
plot(tscl.intersection1, '-xb', 'Displayname', 'Intersection 1')
```

You can see that data points have been interpolated at half-hour intervals, and that Intersection 1 uses zero-order-hold interpolation, while the other two members use linear interpolation.

Maintain the graph in the figure while you add the other two members to the plot. Because the `plot` method suppresses the axis labels while `hold` is on, also add a legend to describe the three series.

```
hold on
plot(tscl.intersection2, '-.xm', 'Displayname', 'Intersection 2')
plot(tscl.intersection3, ':xr', 'Displayname', 'Intersection 3')
legend('show', 'Location', 'NorthWest')
```

### Adding a Data Sample to a Time Series Collection Object

This portion of the example illustrates how to add a data sample to a `tscollection`.

Add a data sample to the `intersection1` collection member at 3.25 hours (i.e., 15 minutes after the hour).

```
tscl = addsampletocollection(tscl, 'time', 3.25, ...
    'intersection1', 5);
```

There are three members in the `tscl` collection, and adding a data sample to one member adds a data sample to the other two members at 3.25 hours. However, because you did not specify the data values for `intersection2` and `intersection3` in the new sample, the missing values are represented by NaNs for these members. To learn how to remove or interpolate missing data values, see “Removing Missing Data” on page 4-13 and “Interpolating Missing Data” on page 4-14.

**tsc1 Data from 2.0 to 3.5 Hours**

Hours	Intersection 1	Intersection 2	Intersection 3
2.0	7	13	11
2.5	7	15	15.5
3.0	14	17	20
3.25	5	NaN	NaN
3.5	14	15	14.5

To view all `intersection1` data (including the new sample at 3.25 hours), type

```
tsc1.intersection1
```

Similarly, to view all `intersection2` data (including the new sample at 3.25 hours containing a NaN value), type

```
tsc1.intersection2
```

**Removing and Interpolating Missing Data**

Time series objects use NaNs to represent missing data. This portion of the example illustrates how to either remove missing data or interpolate values for it by using the interpolation method you specified for that time series. In “Adding a Data Sample to a Time Series Collection Object” on page 4-12, you added a new data sample to the `tsc1` collection at 3.25 hours.

As the `tsc1` collection has three members, adding a data sample to one member added a data sample to the other two members at 3.25 hours. However, because you did not specify the data values for the `intersection2` and `intersection3` members at 3.25 hours, they currently contain missing values, represented by NaNs.

**Removing Missing Data**

Find and remove the data samples containing NaN values in the `tsc1` collection.

```
tsc1 = delsamplefromcollection(tsc1, 'index', ...
    find(isnan(tsc1.intersection2.Data)));
```

This command searches one `tscollection` member at a time—in this case, `intersection2`. When a missing value is located in `intersection2`, the data at that time is removed from *all* members of the `tscollection`.

---

**Note** Use dot-notation syntax to access the `Data` property of the `intersection2` member in the `tscl` collection:

```
tscl.intersection2.Data
```

For a complete list of `timeseries` properties, see “Time Series Properties” on page 4-17.

---

### Interpolating Missing Data

For the sake of this example, reintroduce NaN values in `intersection2` and `intersection3`.

```
tscl = addsamplecollection(tscl, 'time', 3.25, ...  
    'intersection1', 5);
```

Interpolate the missing values in `tscl` using the current time vector (`tscl.Time`).

```
tscl = resample(tscl, tscl.Time);
```

This replaces the NaN values in `intersection2` and `intersection3` by using linear interpolation—the default interpolation method for these time series.

---

**Note** Dot notation `tscl.Time` is used to access the `Time` property of the `tscl` collection. For a complete list of `tscollection` properties, see “Time Series Collection Properties” on page 4-19.

---

To view `intersection2` data after interpolation, for example, type

```
tscl.intersection2
```

### New tsc1 Data from 2.0 to 3.5 Hours

Hours	Intersection 1	Intersection 2	Intersection 3
2.0	7	13	11
2.5	7	15	15.5
3.0	14	17	20
3.25	5	16	17.3
3.5	14	15	14.5

### Removing a Time Series from a Time Series Collection

Remove the `intersection3` time series from the `tscollection` object `tsc1`.

```
tsc1 = removets(tsc1, 'intersection3')
```

Two time series as members in the collection are now listed.

### Displaying Time Vector Values as Date Strings

This portion of the example illustrates how to control the format in which numerical time vector display, using MATLAB date strings. For a complete list of the MATLAB date-string formats supported for `timeseries` and `tscollection` objects, see the definition of time vector definition in the `timeseries` reference page.

To use date strings, you must set the `StartDate` field of the `TimeInfo` property. All values in the time vector are converted to date strings using `StartDate` as a reference date.

Suppose the reference date occurs on December 25, 2009.

```
tsc1.TimeInfo.Units = 'hours';
tsc1.TimeInfo.StartDate = '25-DEC-2009 00:00:00';
```

Similarly to what you did with the `count1`, `count2`, and `count3` time series objects, set the data units to of the `tsc1` members to the string `'car count'`.

```
tsc1.intersection1.DataInfo.Units = 'car count';
tsc1.intersection2.DataInfo.Units = 'car count';
```

### Plotting Time Series Collection Members

To plot data in a time series collection, you plot its members one at a time.

First graph `tscl` member `intersection1`.

```
hold off
plot(tscl.intersection1);
```

When you plot a member of a time series collection, its time units display on the x-axis and its data units display on the y-axis. The plot title is displayed as 'Time Series Plot:<member name>'.  
Time Series Plot:intersection1

If you use the same figure to plot a different member of the collection, no annotations display. The time series `plot` method does not attempt to update labels and titles when `hold` is on because the descriptors for the series can be different.

Plot `intersection1` and `intersection2` in the same figure. Prevent overwriting the plot, but remove axis labels and title. Add a legend and set the `DisplayName` property of the line series to label each member.

```
plot(tscl.intersection1, '-xb', 'Displayname', 'Intersection 1')
hold on
plot(tscl.intersection2, '-.xm', 'Displayname', 'Intersection 2')
legend('show', 'Location', 'NorthWest')
```

The plot now includes the two time series in the collection: `intersection1` and `intesection2`. Plotting the second graph erased the labels on the first graph.

Finally, change the date strings on the x-axis to `hours` and plot the two time series collection members again with a legend.

Specify time units to be `'hours'` for the collection.

```
tscl.TimeInfo.Units = 'hours';
```

Specify the format for displaying time.

```
tscl.TimeInfo.Format = 'HH:MM';
```

Recreate the last plot with new time units.

```
hold off
plot(tscl.intersection1, '-xb', 'Displayname', 'Intersection 1')
% Prevent overwriting plot, but remove axis labels and title.
hold on
plot(tscl.intersection2, '-.xm', 'Displayname', 'Intersection 2')
```

```
legend('show','Location','NorthWest')

% Restore the labels with the |xlabel| and |ylabel| commands and overlay a
% data grid.
xlabel('Time (hours)')
ylabel('car count')
grid on
```

For more information on plotting options for time series, see `timeseries`.

## Time Series Constructor

Before implementing the various MATLAB functions and methods specifically designed to handle time series data, you must create a `timeseries` object to store the data. See `timeseries` for the `timeseries` object constructor syntax.

For an example of using the constructor, see “Creating Time Series Objects” on page 4-6.

### Time Series Properties

See `timeseries` for a description of all the `timeseries` object properties. You can specify the `Data`, `IsTimeFirst`, `Name`, `Quality`, and `Time` properties as input arguments in the constructor. To assign other properties, use the `set` function or dot notation.

---

**Note** To get property information from the command line, type `help timeseries/tsprops` at the MATLAB prompt.

---

For an example of editing `timeseries` object properties, see “Modifying Time Series Units and Interpolation Method” on page 4-9.

### Time Series Methods

For a description of all the time series methods, see `timeseries`.

## Time Series Collection Constructor

- “Introduction” on page 4-18
- “Time Series Collection Constructor Syntax” on page 4-18

- “Time Series Collection Properties” on page 4-19
- “Time Series Collection Methods” on page 4-20

### **Introduction**

The MATLAB object, called `tscollection`, is a MATLAB variable that groups several time series with a common time vector. The `timeseries` objects that you include in the `tscollection` object are called *members* of this collection, and possess several methods for convenient analysis and manipulation of `timeseries`.

### **Time Series Collection Constructor Syntax**

Before you implement the MATLAB methods specifically designed to operate on a collection of `timeseries` objects, you must create a `tscollection` object to store the data.

The following table summarizes the syntax for using the `tscollection` constructor. For an example of using this constructor, see “Creating Time Series Collection Objects” on page 4-10.



### Time Series Collection Syntax Descriptions

Syntax	Description
<code>tsc = tscollection(ts)</code>	<p>Creates a <code>tscollection</code> object <code>tsc</code> that includes one or more <code>timeseries</code> objects.</p> <p>The <code>ts</code> argument can be one of the following:</p> <ul style="list-style-type: none"> <li>• Single <code>timeseries</code> object in the MATLAB workspace</li> <li>• Cell array of <code>timeseries</code> objects in the MATLAB workspace</li> </ul> <p>The <code>timeseries</code> objects share the same time vector in the <code>tscollection</code>.</p>
<code>tsc = tscollection(Time)</code>	<p>Creates an empty <code>tscollection</code> object with the time vector <code>Time</code>.</p> <p>When time values are date strings, you must specify <code>Time</code> as a cell array of date strings.</p>
<code>tsc = tscollection(Time, TimeSeries, 'Parameter', Value, ...)</code>	<p>Optionally enter the following parameter-value pairs after the <code>Time</code> and <code>TimeSeries</code> arguments:</p> <ul style="list-style-type: none"> <li>• <code>Name</code> (see “Time Series Collection Properties” on page 4-19)</li> </ul>

### Time Series Collection Properties

This table lists the properties of the `tscollection` object. You can specify the `Name`, `Time`, and `TimeInfo` properties as input arguments in the `tscollection` constructor.

**Time Series Collection Property Descriptions**

Property	Description
Name	<p><code>tscollection</code> object name entered as a string. This name can differ from the name of the <code>tscollection</code> variable in the MATLAB workspace.</p>
Time	<p>A vector of time values.</p> <p>When <code>TimeInfo.StartDate</code> is empty, the numerical <code>Time</code> values are measured relative to 0 in specified units. When <code>TimeInfo.StartDate</code> is defined, the time values represent date strings measured relative to <code>StartDate</code> in specified units.</p> <p>The length of <code>Time</code> must match either the first or the last dimension of the <code>Data</code> property of each <code>tscollection</code> member.</p>
TimeInfo	<p>Uses the following fields to store contextual information about <code>Time</code>:</p> <ul style="list-style-type: none"> <li>• <code>Units</code> — Time units with the following values: 'weeks', 'days', 'hours', 'minutes', 'seconds', 'milliseconds', 'microseconds', and 'nanoseconds'</li> <li>• <code>Start</code> — Start time</li> <li>• <code>End</code> — End time (read-only)</li> <li>• <code>Increment</code> — Interval between two subsequent time values. The increment is <code>NaN</code> when times are not uniformly sampled.</li> <li>• <code>Length</code> — Length of the time vector (read-only)</li> <li>• <code>Format</code> — String defining the date string display format. See the MATLAB <code>datestr</code> function reference page for more information.</li> <li>• <code>StartDate</code> — Date string defining the reference date. See the MATLAB <code>setabstime</code> function reference page for more information.</li> <li>• <code>UserData</code> — Stores any additional user-defined information</li> </ul>

**Time Series Collection Methods**

- “General Time Series Collection Methods” on page 4-21

- “Data and Time Manipulation Methods” on page 4-21

### General Time Series Collection Methods

Use the following methods to query and set object properties, and plot the data.

#### Methods for Querying Properties

Method	Description
get	Query <code>tscollection</code> object property values.
isempty	Evaluate to true for an empty <code>tscollection</code> object.
length	Return the length of the time vector.
plot	Plot the time series in a collection.
set	Set <code>tscollection</code> property values.
size	Return the size of a <code>tscollection</code> object.

#### Data and Time Manipulation Methods

Use the following methods to add or delete data samples, and manipulate the `tscollection` object.

**Methods for Manipulating Data and Time**

<b>Method</b>	<b>Description</b>
<code>addts</code>	Add a <code>timeseries</code> object to a <code>tscollection</code> object.
<code>addsampletocollection</code>	Add data samples to a <code>tscollection</code> object.
<code>delsamplefromcollection</code>	Delete one or more data samples from a <code>tscollection</code> object.
<code>getabstime</code>	Extract a date-string time vector from a <code>tscollection</code> object into a cell array.
<code>getsamplusingtime</code>	Extract data samples from an existing <code>tscollection</code> object into a new <code>tscollection</code> object.
<code>gettimeseriesnames</code>	Return a cell array of time series names in a <code>tscollection</code> object.
<code>horzcat</code>	Horizontal concatenation of <code>tscollection</code> objects. Combines several <code>timeseries</code> objects with the same time vector into one time series collection.
<code>removets</code>	Remove one or more <code>timeseries</code> objects from a <code>tscollection</code> object.
<code>resample</code>	Select or interpolate data in a <code>tscollection</code> object using a new time vector.
<code>setabstime</code>	Set the time values in the time vector of a <code>tscollection</code> object as date strings.
<code>settimeseriesnames</code>	Change the name of the selected <code>timeseries</code> object in a <code>tscollection</code> object.
<code>vertcat</code>	Vertical concatenation of <code>tscollection</code> objects. Joins several <code>tscollection</code> objects along the time dimension.